ON BUILDING A SYSTEM

An axiom (or rule) is a statement which is accepted as true just for the sake of progress. One never attempts to prove an axiom. Instead, its truth is assumed for the sake of the argument. Such statements are also called "postulates" or "assumptions."

An axiom set is a collection of two (2) or more axioms all of which are assumed to be true at the same time.

A statement which is true in a particular system but which is not an axiom is called a theorem. The word "true" in this context means that it has been "worked out" (explained, defended, proved) using the axioms as a foundation or starting point. A theorem then, is a statement the truth of which must be explained or upheld using axioms, definitions, or other theorems which have already been proved. (A definition is a special kind of axiom. More on this later.)

A proof of a theorem is an explanation or argument showing how the statement follows logically from axioms or from earlier statements which have already been proved. Complete exercises 2-12 through 2-15.

Exercise 2-12 According to the above, a proof is a kind of explanation. Contrast a proof with the kind of explanation necessary to explain why there is smog in the Los Angeles Basin.

Exercise 2-13 State at least two of the theorems of System 1.

Exercise 2-14 State at least three theorems of System 2.

Exercise 2-15 Prove that in System 2 there are at least four members and at least two committees. (Hint: Look at exercise 2-3)

MORE ON BUILDING A SYSTEM

Recall that a deductive system is the entire collection of axioms (the axiom set), all theorems deduced from the axioms, and all versions of the model.

Exercise 2-16 How many deductive systems have you studied so far in this course?

The objects of a system are the "things" that the system deals with. In geometry, the objects of the systems are usually points, lines, and planes.

The relations of a system are descriptions of how objects are located with respect to each other. For instance, three points can be related to each other by saying that one lies "between" the other two. A point can be related to a line by saying that the point "lies on" the line. The same situation can be described by saying that the line "goes through" the point or that the line "contains" the point.

A meta-axiom is a statement which is assumed to be true about all axiom sets or about all deductive systems and their models.