Suppose $T: \mathbb{E}^2 \to \mathbb{E}^2$ is represented by

$T(\bar{x}) = A\bar{x} = \bar{x}'$ where A is a 3x3 matrix

(1) and $\bar{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. We denote $\bar{x}'$ by $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$.

Given a line u represented by $[u_1\ u_2\ u_3]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ or simply by $[u_1\ u_2\ u_3]$ we seek the

(2) image of u under a transformation S. Let's denote that image by $uS = u'$.

(3) Since u, and u' are lines $u\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$ and $u'\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$

(4) So, $u\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = u'\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$. by (3)

(5) By (1) $A\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$

(6) So $u\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = u'A\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = (u'A)\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ By (5), (4)

(7) So, $u = u'A$ by (6)

(8) Also $uA^{-1} = (u'A)A^{-1} = u'(AA^{-1}) = u'$

(9) Hence $uA^{-1} = u'$ (actually ku' for some $k \in \mathbb{R}$)

The line transformation matrix is the inverse of the point transformation matrix.