Theorem: Distance is invariant under the transformation of rotation.

Given two points $\bar{x} = \begin{bmatrix} a \\ b \\ 1 \end{bmatrix}$ and $\bar{y} = \begin{bmatrix} c \\ d \\ 1 \end{bmatrix}$ the distance between them is given by

$$d(\bar{x}, \bar{y}) = \sqrt{(a-c)^2 + (b-d)^2}$$

We know a rotation about the origin is given by the transformation matrix

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The image of the points under the rotation is given by the expressions

$$\bar{x}' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = \begin{bmatrix} a(\cos\theta) - b(\sin\theta) \\ a(\sin\theta) + b(\cos\theta) \\ 1 \end{bmatrix}$$

$$\bar{y}' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} c(\cos\theta) - d(\sin\theta) \\ c(\sin\theta) + d(\cos\theta) \\ 1 \end{bmatrix}$$

The distance of the two points,

$$d(\bar{x}', \bar{y}') = \sqrt{(a-c)^2(\sin^2\theta + \cos^2\theta) + (b-d)^2(\sin^2\theta + \cos^2\theta}$$

$$= \sqrt{(a-c)^2 1 + (b-d)^2 1} = d(\bar{x}, \bar{y})$$

The distances are the same before and after rotation.
Therefore, distance is invariant under the transformation of rotation.