Discrete Models

Model 1: $p_{t+1} - p_t = A$ for $t = 0, 1, 2, 3, \ldots$ and p_0 is given.

$p_{t+1} = p_t + A$ for $t = 0, 1, 2, 3, \ldots$

$p_1 = p_0 + A$
$p_2 = p_1 + A = (p_0 + A) + A = p_0 + 2A$
$p_3 = p_2 + A = (p_0 + 2A) + A = p_0 + 3A$
$p_4 = p_3 + A = (p_0 + 3A) + A = p_0 + 4A$
:
$P_n = p_{n-1} + A = (p_0 + (n-1)A) + A = p_0 + nA$ for $n = 1, 2, 3, \ldots$

Model 2: $p_{t+1} - p_t = rp_t$ for $t = 0, 1, 2, 3, \ldots$ and p_0 is given.

$p_{t+1} = p_t + rp_t = (1 + r)p_t$ for $t = 0, 1, 2, 3, \ldots$

$p_1 = p_0 + rp_0 = (1 + r)p_0$
$p_2 = p_1 + rp_1 = (1 + r)p_1 = (1 + r)(1 + r)p_0 = (1 + r)^2 p_0$
$p_3 = p_2 + rp_2 = (1 + r)p_2 = (1 + r)(1 + r)^2 p_0 = (1 + r)^3 p_0$
$p_4 = p_3 + rp_3 = (1 + r)p_3 = (1 + r)(1 + r)^3 p_0 = (1 + r)^4 p_0$
:
$P_n = p_{n-1} + rp_{n-1} = (1 + r)p_{n-1} = (1 + r)(1 + r)^{n-1} p_0 = (1 + r)^n p_0$ for $n = 1, 2, 3, \ldots$

Model 3: $p_{t+1} - p_t = cp_t(1 - p_t/M)$ for $t = o, 1, 2, 3, \ldots$ and p_0 is given.

Continuous Models

Model 1:

$\frac{dp}{dt} = A$ **and *p(0)* is given.**

$p(t) = At + p(0)$

Model 2:

$\frac{dp}{dt} = rp$ **and *p(0)* is given.**

Separating variables,

$$\frac{dp}{p} = rdt .$$

Integrating we get $\ln(p) = rt + c$. Thus, $p(t) = e^{rt + c} = e^{c}(e^{rt})$

Hence, employing the initial condition,

$p(t) = p(0)e^{rt}$.

Model 3:

$\frac{dp}{dt} = rp(1 - \frac{p}{M})$ ***and p(0)* is given.**

Separating variables,

$$\frac{dp}{p(1 - \frac{p}{M})} = rdt .$$

Now, integrate both sides and apply the initial condition.