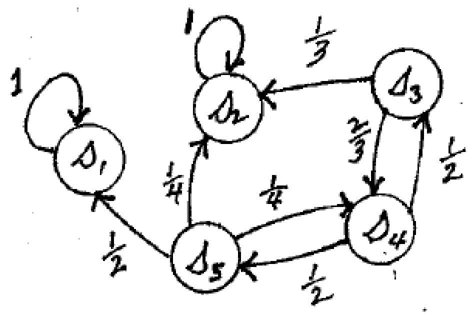
Markov Chains – Discussion Continued

Consider the absorbing MCP represented by the weighted digraph below.



Our convention is that the *canonical form* of the transition matrix of an absorbing MCP has states ordered so that the absorbing states are listed first. With this convention, the transition matrix **P** will have the form.

$$\mathbf{P} = \begin{bmatrix} I & O \\ R & Q \end{bmatrix}.$$

We have seen that

$$\lim_{m\to\infty}\mathbf{P}^{\mathbf{m}} = \begin{bmatrix} I & O \\ NR & O \end{bmatrix}$$ where $\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$ and we call **N** the *fundamental matrix* for the process with matrix **P**.

Identify the canonical form for the transition matrix for the absorbing MCP represented by the diagram above.

$$\mathbf{P} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

In this example the fundamental matrix for the MCP is

$$\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

So, $$\lim_{m\to\infty}\mathbf{P}^{\mathbf{m}} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

There are some important results we gain from the fundamental matrix **N**.

n_{ij}, the ij-entry of **N**, is the expected number of visits to the j^{th} nonabsorbing state given the process began in the i^{th} nonabsorbing state and continued until an absorbing state was reached. (See proof on p. 137) The sum of the entries in the i^{th} row of **N** is the expected number of transitions before an absorbing state is reached if the process began in the i^{th} nonabsorbing state.

The matrix **A** = **NR** also provides significant information. The *ij-entry* of **A**, a_{ij}, is the probability that a process that began in the i^{th} nonabsorbing state is absorbed in the j^{th} absorbing state.

It is important to remember that in this discussion that the references to states *i* and *j* refer to the states of the matrix in canonical form.

Respond to the following questions about the absorbing MCP with transition matrix **P.**

1. Suppose the process begins in s_4 and runs until reaching an absorbing state.

    a. What is the expected number of times the process will be in s_5 prior to absorption?

    b. What is the expected number of transitions prior to absorption?

    c. What is the probability of absorption into s_2?

2. Suppose the process begins in s_3. What is the probability the process is in s_5 prior to absorption?