

Given a group of people and a set of alternatives on which the people have opinions.

We suppose each individual has a *preference ranking* on the set of alternatives.

A set of individual preference rankings is called a *group preference schedule*.

Given a group preference schedule, one goal will be to construct a *group preference ranking* if possible.

We will assume that individual preferences are *transitive*, and we will require that group preferences be transitive.

We will ignore the strength of preferences.

We require that individual preferences are well defined. (We ignore indifference.)

Example 1: We have a group of three people A, B, C and a set of three alternatives X, Y, Z. Suppose the individual preference rankings are as follows:

	A	B	C
1 st	X	Y	Z
2 nd	Y	Z	X
3 rd	Z	X	Y

Here A prefers X to Y to Z. We denote this by $X > Y > Z$.

Our initial approach will be to try *pairwise comparisons* selecting the preferred alternative of a pair by simple majority vote. If this leads to a transitive group preference ranking, then we call the one at the top of the group list the *most preferred alternative*.

	A	B	C
1 st	X	Y	Z
2 nd	Y	Z	X
3 rd	Z	X	Y

Making the pairwise comparisons it looks like the group preferences should be $X > Y > Z > X$. As noted above, we will consider this non-transitive, or cyclic, behavior unacceptable.

Example 2: The individual preference rankings are as follows.

	A	B	C
1 st	X	Y	Z
2 nd	Y	Z	Y
3 rd	Z	X	X

Employing pairwise comparisons we have a group preference ranking $Y > Z > X$ but no majority winner.

Example 3: We now consider a sequential voting scheme where a most preferred alternative is to first determined by first selecting between any two alternatives and then selecting between the winner of the first step and a third alternative, and so on. What happens when we apply this sequential voting approach to the preference rankings in Example 1.

Our next approach will be to assigning points to votes on the basis of their relative rankings and determining a group preference ranking by adding the points assigned to each alternative by all the individuals.

Example 4:

Points	A	B	C
4	X	Y	W
3	Y	Z	X
2	Z	W	Y
1	W	X	Z

Alternative	Sum of Points
X	
Y	
Z	
W	

We have a group preference ranking of $Y > X > W > Z$.

Example 5: Apply the method of Example 4 to the following preference rankings.

A	B	C	D	E	F	G
X	Y	W	X	W	Y	W
Y	Z	X	Y	X	Z	X
Z	W	Y	Z	Y	W	Y
W	X	Z	W	Z	X	Z

A	B	C	D	E	F	G
X	Y	W	X	W	Y	W
Y	W	X	Y	X	W	X
W	X	Y	W	Y	X	Y

Summary of Desired Properties of a Group Preference Ranking

1. The decision rule must produce a transitive group preference ranking.
2. If alternative X is preferred to alternative Y by every member of the group, then the decision process produces a group preference ranking in which X is preferred to Y.
3. Let V be a subset of alternatives. The decision process should determine a group preference ranking in which the relative rankings of the alternatives in V depends only on the relative ranking of those alternatives by the individuals.
4. There is no individual I such that for every preference schedule, the decision process yields a group preference ranking that is the same as I's.

Example 6: Apply the method of Example 4 to the following preference rankings.

A	B	C	D	E
X	X	Y	Z	W
Z	Y	X	W	X
Y	Z	W	X	Y
W	W	Z	Y	Z

A	B	C	D	E
Z	X	Y	W	W
X	W	X	Z	Y
Y	Y	W	Y	Z
W	Z	Z	X	X

Compare the relative rankings of X and W.