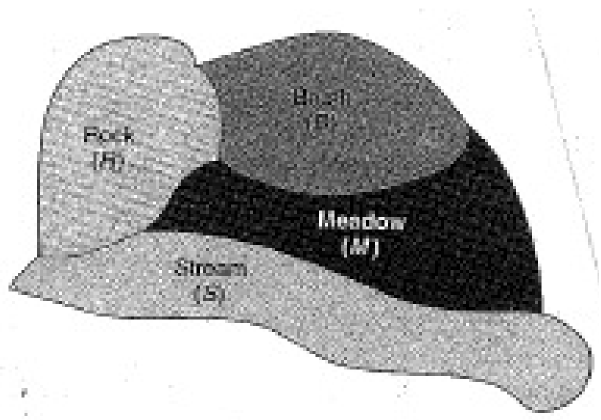
A Marmot's Range Investigated

http://www.torontozoo.com/images/Con-mam-marmot.jpg

A locale (See the figure below.) consists off rocks (R), scrub brush (B), open meadow (M), and a stream (S). This locale is home for a marmot. We seek to model the marmot's movement over time by noting its location at sequential observations and then forming a mathematical system that represents those movements in an appropriate way.



We might be able to represent a sequence of observations by a sequence of letters (locations) and numbers (occupancy times):

$$R \xrightarrow{34.2} B \xrightarrow{12.3} R \xrightarrow{2.4} S \xrightarrow{10.5} M \xrightarrow{20.7} B$$

We will elect to concentrate on the location alone ignoring occupancy times. Depending on the criteria we use to make observations, locations may or may not appear successively in a sequence of observations. So a sequence of locations may be represented as RRBRSMMB or by a *sample path* diagram
R→R→B→R→S→M→M→B.

For our example, we assume that the marmot is observed periodically and also whenever it moves from one subarea to another. We also assume that it is twice as likely to remain where it is as it is to move to any one of the other areas available to it. Under those assumptions, we assign probabilities to the various moves.

		Location after Move			
		R	B	M	S
Location before Move	R	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
	B	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0
	M	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{1}{9}$
	S	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{2}{3}$

Suppose we associate the locations with states s_1, s_2, s_3, s_4 in a MCP and the probabilities in the table above as transition probabilities p_{ij} giving the probability of moving from s_i into s_j on the next observation. Our *transition matrix* is a *4 x 4* matrix **P** whose *ij*-entry is p_{ij}.

Definitions, Notation, and Results

We let $\mathbf{P}(m) = [p_{ij}(m)]$ be the matrix whose *ij*-entry is the probability of making a transition from s_i to s_j in *m* steps (trials, observations). Of course $\mathbf{P}(1) = \mathbf{P}$.

A state vector is a probability vector that describes the status of a MCP at an observation, and the state vectors at two successive observations are related in a simple way. If $\mathbf{x}(m)$ and $\mathbf{x}(m+1)$ denote the state vectors at the m^{th} and $(m+1)^{st}$ observations, then $\mathbf{x}(m+1) = \mathbf{x}(m)\mathbf{P}$.

If the *initial state vector* is $\mathbf{x}(0)$, it is easy to show that $\mathbf{x}(k) = \mathbf{x}(0)\mathbf{P}^k$ for k = 1, 2, 3, …

It follows that $\mathbf{P}(k) = \mathbf{P}^k$.

Calculate $\mathbf{P}^{20}$ interpret its significance in terms of this model.