

Seasonal Changes in a Raised Mine

Find a fixed probability vector $\bar{p} = [p_1 \ p_2 \ p_3 \ p_4]$
Such that

$$[p_1 \ p_2 \ p_3 \ p_4] \begin{bmatrix} 0.65 & 0.29 & 0.06 & 0.00 \\ 0.30 & 0.33 & 0.30 & 0.07 \\ 0.00 & 0.28 & 0.69 & 0.03 \\ 0.00 & 0.40 & 0.20 & 0.40 \end{bmatrix} = [p_1 \ p_2 \ p_3 \ p_4]$$

$$\begin{cases} 0.65p_1 + 0.30p_2 & = p_1 \\ 0.29p_1 + 0.33p_2 + 0.28p_3 + 0.40p_4 & = p_2 \\ 0.06p_1 + 0.30p_2 + 0.69p_3 + 0.20p_4 & = p_3 \\ 0.07p_2 + 0.03p_3 + 0.40p_4 & = p_4 \\ p_1 + p_2 + p_3 + p_4 & = 1 \end{cases}$$

$$\begin{cases} -0.35p_1 + 0.30p_2 & = 0 \\ 0.29p_1 - 0.67p_2 + 0.28p_3 + 0.40p_4 & = 0 \\ 0.06p_1 + 0.30p_2 - 0.31p_3 + 0.20p_4 & = 0 \\ 0.07p_2 + 0.03p_3 - 0.60p_4 & = 0 \\ p_1 + p_2 + p_3 + p_4 & = 1 \end{cases}$$

$$\begin{cases} p_1 & = 0.261 \\ p_2 & = 0.304 \\ p_3 & = 0.380 \\ p_4 & = 0.055 \end{cases}$$

So, the \bar{p} we seek is

$$[0.261 \ 0.304 \ 0.380 \ 0.055] = \bar{p}$$

Remark

$$\left. \begin{bmatrix} 0.65 & 0.29 & 0.06 & 0.00 \\ 0.30 & 0.33 & 0.30 & 0.07 \\ 0.00 & 0.28 & 0.69 & 0.03 \\ 0.00 & 0.40 & 0.20 & 0.40 \end{bmatrix} \right\}^n \rightarrow \begin{bmatrix} 0.261 & 0.304 & 0.380 & 0.055 \\ 0.261 & 0.304 & 0.380 & 0.055 \\ 0.261 & 0.304 & 0.380 & 0.055 \\ 0.261 & 0.304 & 0.380 & 0.055 \end{bmatrix}$$

How long will it take to move from s_1 (bog) to s_4 (grazed)?

$$T = \left(\begin{array}{ccc|c} 0.65 & 0.29 & 0.06 & 0.00 \\ 0.30 & 0.33 & 0.30 & 0.07 \\ 0.00 & 0.28 & 0.69 & 0.03 \\ \hline 0.00 & 0.00 & 0.00 & 1.00 \end{array} \right) =$$

$$Q = \begin{pmatrix} 0.65 & 0.29 & 0.06 \\ 0.30 & 0.33 & 0.30 \\ 0.00 & 0.28 & 0.69 \end{pmatrix}$$

$$I_3 - Q = \begin{pmatrix} 0.35 & -0.29 & -0.06 \\ -0.30 & 0.67 & -0.30 \\ -0.00 & -0.28 & 0.31 \end{pmatrix}$$

$$N = (I_3 - Q)^{-1} \approx \begin{bmatrix} 10.962 & 9.455 & 11.272 \\ 8.241 & 9.615 & 10.899 \\ 7.444 & 8.684 & 13.070 \end{bmatrix}$$

Mean number of transitions to reach s_4 from s_1 is about 31.7.

mean number of transitions to reach s_4 from s_2 is about 28.8

mean number of transitions to reach s_4 from s_3 is about 29.1.