Assumptions in the real model:

- The lynx species is totally dependent on the hare species as its only food supply.
- The hare species has an unlimited food supply and there exists no threat to its growth other than the specific predator.
- The rate at which lynx encounter hares is jointly proportional to the sizes of the two populations.
- A fixed proportion of encounters leads to the death of hares.
- The energy to support growth of the lynx population is proportional to deaths of prey.

A Mathematical Model for Predator-Prey Problem – Continuous Version

Undefined terms: hare, lynx

Definitions: Time is any nonnegative real number

Axioms:

A_1: The number of lynx present at time t, denoted by L(t), is a differentiable function. The number of hares present at time t, denoted by H(t), is a differentiable function.

A_2: *H(0)* and *L(0)* are positive integers.

A_2: There exists positive constants r, s, a, and b such that

$$(*)\quad \begin{aligned} \frac{dH}{dt} &= rH - aHL \\ \frac{DL}{dt} &= -sL + bHL \end{aligned}$$

A Mathematical Model for Predator-Prey Problem – Discrete Version

Undefined terms: hare, lynx

Definitions: Time is any nonnegative integer
The number of lynx present at time t is denoted by L(t).
The number of hares present at time t is denoted by H(t).

A_1: H(0) and L(0) and are positive integers.

A_2: There exists positive constants r, s, a, and b such that

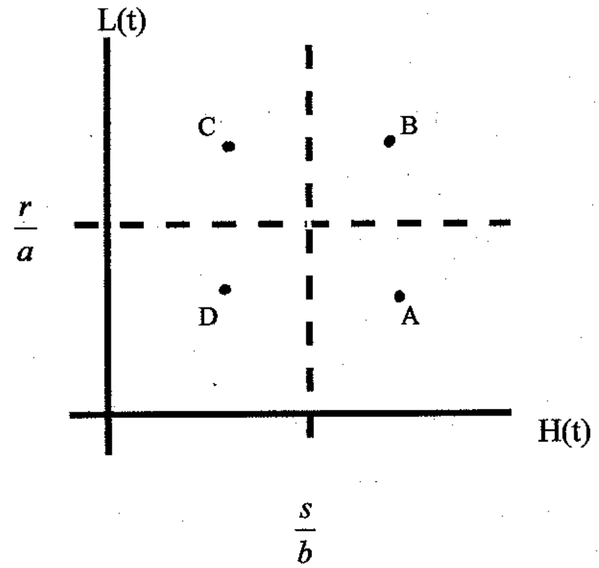
$$(**)\quad \begin{aligned} H(n+1) - H(n) &= rH(n) - aH(n)L(n) \\ L(n+1) - L(n) &= -sL(n) + bH(n)L(n) \end{aligned}$$

Analysis of Qualitative Behavior of Model in Continuous Case

Equations (*) can be rewritten as

$$(***)\qquad \begin{aligned} \frac{dH}{dt} &= aH\left(\frac{r}{a} - L\right) \\ \frac{dL}{dt} &= bL\left(H - \frac{s}{b}\right) \end{aligned}$$

We note that if L(t) = (r/a) and H(t) = (s/b) for some time, then we are at a stable point for the system (*). What can we learn about what occurs in the model as we move away from the stable point?



Where do we move next should we arrive at point A?

Where do we move next should we arrive at point B?

Where do we move next should we arrive at point C?

Where do we move next should we arrive at point D?