Assign #1 Notes

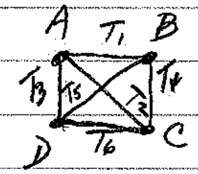
P.23 #1 Show that Model 2 on page 17 is a model for axiom system Σ_1

players: A, B, C, D

teams: $T_1 = \{A, B\}$, $T_2 = \{A, C\}$, $T_3 = \{A, D\}$

$T_4 = \{B, C\}$, $T_5 = \{B, D\}$, $T_6 = \{C, D\}$

We will us the picture below to verify that we have a model for Σ_1. Dots represent players and segments represent teams.



A_1 is clearly satified as each team has exactly two players.

A_2 is clearly satisfied as there are four players.

A_3 is illustrated in the diagram. Each pair of players is associated with a unique team.

A_4 is verified by considering the 12 triples below.

T_1, C, T_6	T_3, B, T_4	T_5, A, T_2
T_1, D, T_4	T_3, C, T_4	T_5, C, T_2
T_2, B, T_5	T_4, A, T_3	T_6, A, T_1
T_2 D, T_5	T_4 D, T_3	T_6, B, T_1

P.23 #2 Prove: In Σ_1 if there are three distinct teams, then there are four distinct teams.

Pf

Note two cases.

let T_1, T_2, T_3 be three distinct teams. Then by the axioms there are three distinct players P_1, P_2, P_3 such that $P_1, P_2 \in T_1$, $P_1 \notin T_2$, $P_3 \in T_2$, $P_3 \notin T_1$.

Consider the unique team containing P_1 and P_3. Either that team is T_3 or it is a fourth team.

So, suppose that team is T_3. In this case $P_2 \notin T_3$ otherwise we would have $T_1 = T_3$. So, there must be a team containing P_2 that is disjoint from T_3. That team will be our fourth team because neither T_1 nor T_2 is disjoint from T_3.

P.23 #3) Look at team $\{A,C\}$ and player B.
$\{B,D\}$ and $\{B,E\}$ are teams disjoint from $\{A,C\}$ containing B. So, A_4 fails.

P.23 #8) Any model must contain exactly one team and exactly five players. So, that is essentially one model.

P.23 #9) a) Since model z is a model for Σ, it must also be a model for Σ' because...
b) There are three essentially different models for Σ' with exactly five players. They are ...

P.23 #13)  can represent a model for Γ
 can represent a model for $\{\Gamma - A_5, \sim A_5\}$.
So, A_5 is independent in Γ