

3.3
16

original matrix

$$\begin{array}{c}
 A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \\
 \left[\begin{array}{ccccc}
 A_1 & 0 & 0 & 1 & 0 & 0 \\
 A_2 & 0 & .4 & 0 & .6 & 0 \\
 A_3 & .2 & .3 & 0 & .5 & 0 \\
 A_4 & 0 & .5 & 0 & .1 & .4 \\
 A_5 & 0 & 0 & 0 & 1 & 0
 \end{array} \right]
 \end{array}$$

We can partition the set of states into two sets of mutually accessible states. First we let S_1 be the set of all states mutually accessible from A_1 .

$$S_1 = \{A_1, A_3\}$$

Now we let S_2 be the set of states mutually accessible from A_2 .

$$S_2 = \{A_2, A_4, A_5\}$$

We reorder the states so the states in S_2 are listed first and form a new transition matrix

$$\begin{array}{c}
 A_2 \quad A_4 \quad A_5 \quad A_1 \quad A_3 \\
 \left[\begin{array}{ccccc}
 S_2 \left\{ \begin{array}{l} A_2 \\ A_4 \\ A_5 \end{array} \right. & \begin{array}{ccccc}
 .4 & .6 & 0 & 1 & 0 & 0 \\
 .5 & .1 & .4 & 1 & 0 & 0 \\
 0 & 1 & 0 & 1 & 0 & 0 \\
 \hline
 S_1 \left\{ \begin{array}{l} A_1 \\ A_3 \end{array} \right. & \begin{array}{ccccc}
 0 & 0 & 0 & 0 & 1 \\
 .3 & .5 & 0 & .2 & 0
 \end{array}
 \end{array}
 \right]
 \end{array}$$

In S_2 each state has period 2.

In S_1 each state has period 2.

This process is not ergodic.
(You can look at powers of this matrix.)

3.3
1d

original matrix

$$\begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \\ \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ .5 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] = \mathbb{T}$$

Let S_i be the set of all states mutually accessible from s_i .

$$S_1 = \{s_1, s_2, s_3, s_4, s_5\} = S \text{ (the set of all states)}$$

✓ So, the process is ergodic and its matrix is already in canonical form. (See defn. 3.5)

The index of each state is 3. Look at the diagonal entries of $\mathbb{T}, \mathbb{T}^2, \mathbb{T}^3$

The process is not regular because there are zeros in all powers of \mathbb{T} .