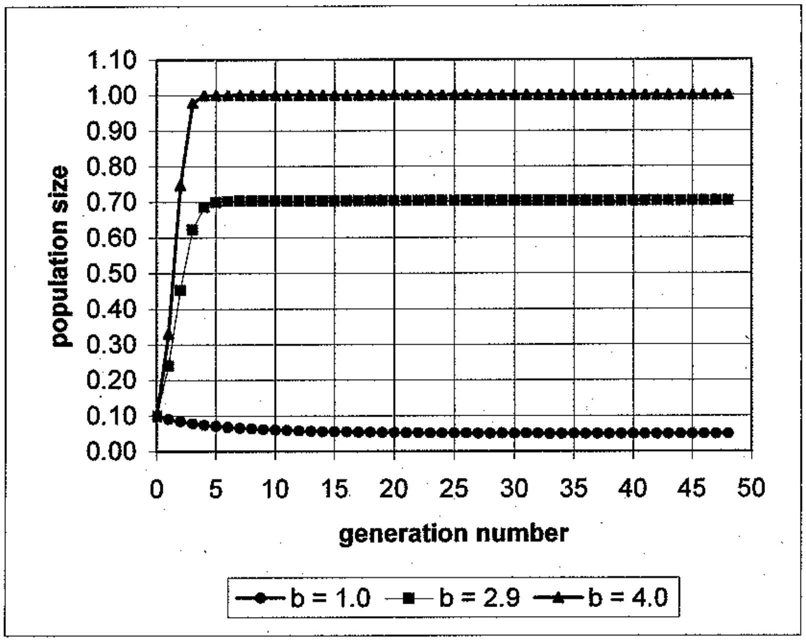
2.2/#9

x(n+1) = bx(n)/[1 + x(n)]^2

√b - 1 =	0.0488	0.7029	1.0000
b =	1.1000	2.9000	4.0000
n	x(n)	x(n)	x(n)
0	0.1000	0.1000	0.1000
1	0.0909	0.2397	0.3306
2	0.0840	0.4523	0.7469
3	0.0787	0.6219	0.9790
4	0.0744	0.6856	0.9999
5	0.0709	0.6998	1.0000
6	0.0680	0.7024	1.0000
7	0.0656	0.7028	1.0000
8	0.0635	0.7029	1.0000
9	0.0618	0.7029	1.0000
10	0.0603	0.7029	1.0000
11	0.0590	0.7029	1.0000
12	0.0578	0.7029	1.0000
13	0.0569	0.7029	1.0000
14	0.0560	0.7029	1.0000
15	0.0552	0.7029	1.0000
16	0.0546	0.7029	1.0000
17	0.0540	0.7029	1.0000
18	0.0534	0.7029	1.0000
19	0.0530	0.7029	1.0000
20	0.0526	0.7029	1.0000
21	0.0522	0.7029	1.0000
22	0.0519	0.7029	1.0000
23	0.0516	0.7029	1.0000
24	0.0513	0.7029	1.0000
25	0.0510	0.7029	1.0000
26	0.0508	0.7029	1.0000
27	0.0506	0.7029	1.0000
28	0.0505	0.7029	1.0000
29	0.0503	0.7029	1.0000
30	0.0502	0.7029	1.0000
31	0.0500	0.7029	1.0000
32	0.0499	0.7029	1.0000
33	0.0498	0.7029	1.0000
34	0.0497	0.7029	1.0000
35	0.0496	0.7029	1.0000
36	0.0495	0.7029	1.0000
37	0.0495	0.7029	1.0000
38	0.0494	0.7029	1.0000
39	0.0494	0.7029	1.0000
40	0.0493	0.7029	1.0000
41	0.0493	0.7029	1.0000
42	0.0492	0.7029	1.0000
43	0.0492	0.7029	1.0000
44	0.0491	0.7029	1.0000
45	0.0491	0.7029	1.0000
46	0.0491	0.7029	1.0000
47	0.0491	0.7029	1.0000
48	0.0490	0.7029	1.0000
49	0.0490	0.7029	1.0000
50	0.0490	0.7029	1.0000
51	0.0490	0.7029	1.0000
52	0.0490	0.7029	1.0000



Find maximum value for x_n.

let $f(x) = \frac{bx}{(1+x)^2}$, $b>0$

$f'(x) = \frac{b(x^2-1)}{(1+x)^4} \Rightarrow f'(x) = 0$ if $x = \pm 1$

$f'(x)$ changes from positive to negative around 1, so $f(1)$ is a maximum

$f(1) = \frac{b}{4}$, so $f(x) \le 1$ provided $b \le 4$.

So, $\{x_n\}$ a population model for $b \le 4$.

Find stable point

$f(x) = x \Rightarrow x = \frac{bx}{(1+x)^2}$

$\Rightarrow x = \sqrt{b} - 1$ ← stable point when $b \ge 1$

$\{x_n\} \to 0$ if $b \le 1$

$\{x_n\} \xrightarrow{\text{monotonic}} \sqrt{b} - 1$ if $1 < b \le 4$

$\{x_n\} \xrightarrow{\text{non-monotonic}} \sqrt{b} - 1$ if $b > 4$