Item #6

The Model (Discrete Case)

Undefined Terms: lynx, hare

Definitions: Time is a non-negative integer
The number of lynx present at time t is $L(t)$
The number of hares present at time t is $H(t)$

Axioms:

A_1: $H(0)$ and $L(0)$ are positive integers

A_2: There exists constants $r, s, a, b,$ and c such that

$$(**)\quad \begin{cases} H(t+1) - H(t) = rH(t) - cH^2(t) - aH(t)L(t) \\ L(t+1) - L(t) = -sL(t) + bL(t)H(t) \end{cases}$$

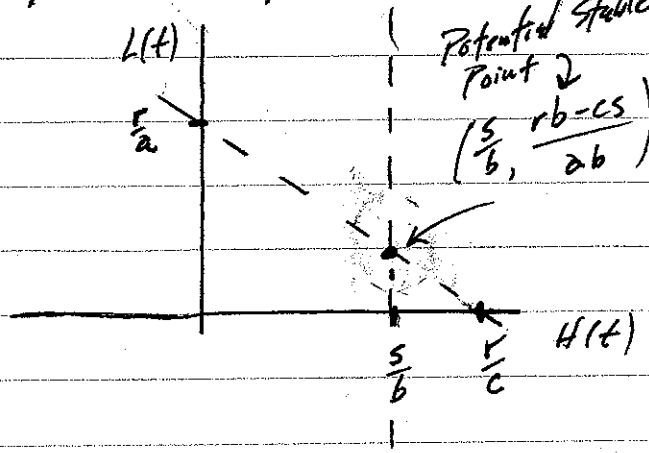
Note the equations could be written as

$$\begin{cases} H(t+1) - H(t) = H(t)\left[r - cH(t) - aL(t)\right] \\ L(t+1) - L(t) = L(t)\left[-s + bH(t)\right] \end{cases}$$

If $L(t) = \left(\frac{r}{a} - \frac{cH(t)}{a}\right)$ and $H(t) = \frac{s}{b}$ then we are at a stable point for the system $(**)$.

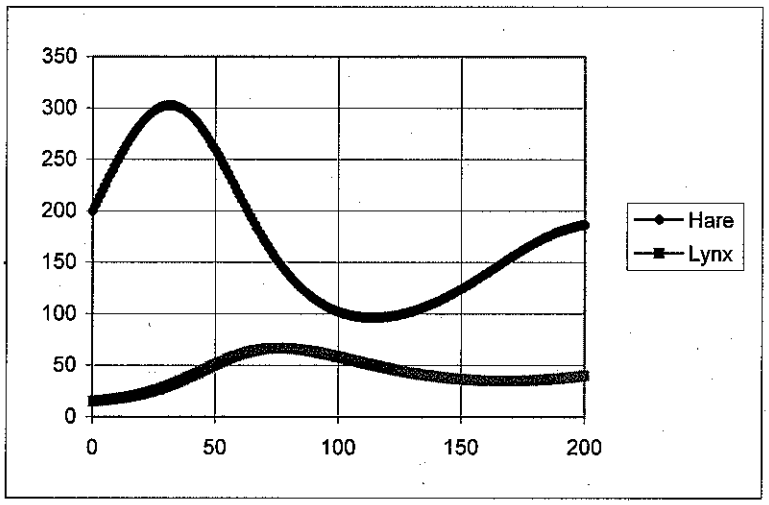
We find a stable point x^* where $x^* = \left(\frac{s}{b}, \frac{rb-cs}{ab}\right)$.

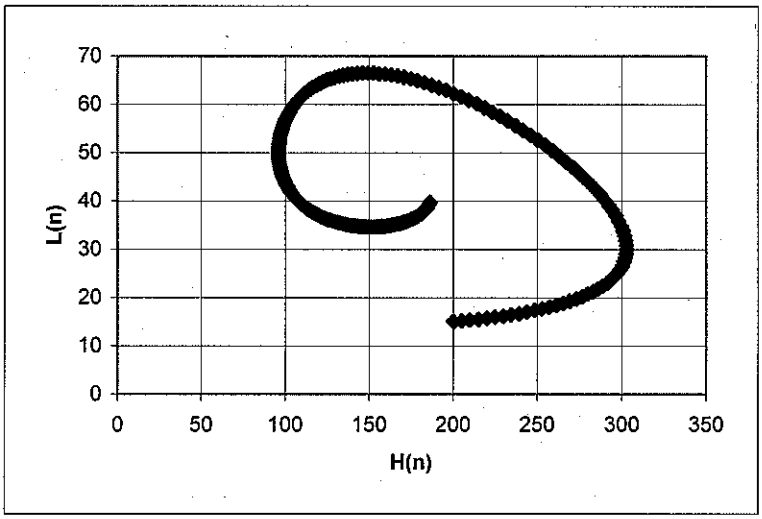
The attached graphs illustrate this situation for a specific set of parameters.



r = 0.06000 s= 0.03000 a= 0.00100 b= 0.00020 s/b= 150 r/a= 60
c= 0.00010 (rb-cs)/ab = 45

n	H(n)	L(n)
0	200	15
1	205	15
2	210	15
3	215	16
4	220	16
5	225	16
6	230	16
7	234	16
8	239	17
9	244	17
10	248	17
11	253	18
12	257	18
13	261	18
14	265	19
15	269	19
16	273	20
17	277	20
18	280	21
19	283	21
20	286	22
21	289	22
22	291	23
23	294	24
24	296	24
25	297	25
26	299	26
27	300	27
28	301	27
29	302	28
30	303	29
31	303	30
32	303	31
33	302	32
34	302	33
35	301	34
36	300	35
37	298	36
38	297	37
39	295	38
40	292	39
41	290	40
42	287	41
43	284	42
44	281	44
45	278	45
46	275	46
47	271	47
48	267	48
49	263	49
50	259	50
51	255	51
52	251	53
53	246	54





r = 0.06000 s= 0.03000 a= 0.00100 b= 0.00020 s/b= 150 r/a= 60
c= 0.00010 (rb-cs)/ab = 45

n	H(n)	L(n)
0	150	45
1	150	45
2	150	45
3	150	45
4	150	45
5	150	45
6	150	45
7	150	45
8	150	45
9	150	45
10	150	45
11	150	45
12	150	45
13	150	45
14	150	45
15	150	45
16	150	45
17	150	45
18	150	45
19	150	45
20	150	45
21	150	45
22	150	45
23	150	45
24	150	45
25	150	45
26	150	45
27	150	45
28	150	45
29	150	45
30	150	45
31	150	45
32	150	45
33	150	45
34	150	45
35	150	45
36	150	45
37	150	45
38	150	45
39	150	45
40	150	45
41	150	45
42	150	45
43	150	45
44	150	45
45	150	45
46	150	45
47	150	45
48	150	45
49	150	45
50	150	45
51	150	45
52	150	45
53	150	45