3.4/49

We define our states as follows
R - marmot in rocks; B - marmot in brush;
MF - marmot in mud flat; M - marmot in marsh;
F - marmot in forest.

a)

$$\begin{array}{c|c} & \begin{array}{ccccc} R & B & MF & M & F \end{array} \\ \begin{array}{c} R \\ B \\ MF \\ M \\ F \end{array} & \begin{pmatrix} .20 & .60 & .20 & 0 & 0 \\ .16 & .20 & .16 & .32 & .16 \\ .1333 & .40 & .20 & .2667 & 0 \\ 0 & .48 & .16 & .20 & .16 \\ 0 & .48 & 0 & .32 & .20 \end{pmatrix} = \mathbb{P} \end{array}$$

b) Given the marmot begins in R, to find the expected number of observations before reaching the marsh we make the marsh (M) an absorbing state. (See remarks on page 144.)

$$\begin{array}{c|c} & \begin{array}{c:cccc} M & R & B & MF & F \end{array} \\ \begin{array}{c} M \\ R \\ B \\ MF \\ F \end{array} & \left(\begin{array}{c:cccc} 1 & 0 & 0 & 0 & 0 \\ \hline 0 & .2 & .6 & .2 & 0 \\ .32 & .16 & .2 & .16 & .16 \\ .2667 & .1333 & .4 & .2 & 0 \\ .32 & 0 & .48 & 0 & .2 \end{array}\right) = \hat{\mathbb{P}} \\ & \begin{array}{cc} \uparrow & \uparrow \\ \hat{R}_{4\times 1} & \hat{Q}_{4\times 4} \end{array} \end{array}$$

expected visits to R prior to absorption in M
expected visits to B prior to absorption " "
" " " MF " " "
" " " F " " "

$$\hat{N} = \left(\mathbb{I}_{4\times 4} - \hat{Q}\right)^{-1} = \begin{array}{c} R \\ B \\ MF \\ F \end{array} \begin{pmatrix} 1.7944 & 2.6136 & 0.8512 & 0.4026 \\ 0.5368 & 2.2047 & 0.5751 & 0.4409 \\ 0.5674 & 1.4378 & 1.6194 & 0.2876 \\ 0.3221 & 1.3228 & 0.3451 & 1.5146 \end{pmatrix}$$

So, the expected value we seek is

$\sum_{j=1}^{4} n_{1j} \doteq 5.0612$. (The expected value we seek)

This result follows from Theorem 3.5, and given the marmot begins in the rock pile, the expected number of observations before reaching the marsh is about 5.06.

3.4 #9 continued

c) Given the marmot begins in R we seek the probability it reaches F without going through M.

In this case we make F and M absorbing.

$$\begin{array}{c|c} & \begin{matrix} M & F & R & B & MF \end{matrix} \\ \begin{matrix} M \\ F \\ R \\ B \\ MF \end{matrix} & \left(\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & .2 & .6 & .2 \\ .32 & .16 & .16 & .2 & .16 \\ .2667 & 0 & .1333 & .4 & .2 \end{array}\right) = \hat{\mathbb{P}} \end{array}$$

$\hat{R}_{3\times1}$ $\quad$ $\hat{Q}_{3\times3}$

$$\hat{N} = \left(\mathbb{I}_{3\times3} - \hat{Q}\right)^{-1} = \begin{pmatrix} 1.7088 & 1.6613 & .7595 \\ .4430 & 1.8196 & .4747 \\ .506 & 1.1866 & 1.6139 \end{pmatrix}$$

$$\hat{N}\hat{R} = \begin{array}{c|c} & \begin{matrix} M & F \end{matrix} \\ \begin{matrix} R \\ B \\ MF \end{matrix} & \begin{pmatrix} .7342 & .2658 \\ .7089 & .2911 \\ .8101 & .1899 \end{pmatrix} \end{array}$$

The probability we seek is 0.2658

This result follows from Theorem 3.6. Since the probability of absorption into state F is 0.2658, that's the probability the marmot visits the forest without going through the marsh, given it starts in the rock pile.