#1 a) Find $\bar{f} = (f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23})$ to minimize $\bar{f} \cdot (50, 80, 40, 50, 40, 40)$ such that $f_{ij} \geq 0$ for all i,j and

$$
\begin{aligned}
f_{11} + f_{12} + f_{13} \qquad\qquad\qquad &= 15 \\
f_{21} + f_{22} + f_{23} &= 15 \\
f_{11} \qquad + f_{21} \qquad\qquad &= 8 \\
f_{12} \qquad + f_{22} \qquad &= 12 \\
f_{13} \qquad + f_{23} &= 10
\end{aligned}
$$

By Thm 2.4

$\bar{f} \cdot \bar{c} = 1760 - 40 f_{22} + 0 f_{23}$ and the constraints reduce to

$$0 \leq f_{22} \leq 12$$
$$0 \leq f_{23} \leq 10$$
$$0 \leq f_{22} + f_{23} \leq 15$$

we take f_{22} as large as possible and f_{23} any feasible value and find our minimum cost is \$1280 and <u>one</u> optimal solution is $f_{11} = 5$; $f_{12} = 0$; $f_{13} = 0$; $f_{21} = 3$; $f_{22} = 12$; $f_{23} = 0$.

#1 d) In this case, by Thm 2.4 we get

$\bar{f} \cdot \bar{c} = 44 - 1 f_{22} + 2 f_{23}$ and the constraints reduce to

$$0 \leq f_{22} \leq 4$$
$$0 \leq f_{22} \leq 3$$
$$3 \leq f_{22} + f_{23} \leq 5$$

Taking f_{22} as large as possible and f_{23} as small as possible our <u>unique</u> optimal solution is $f_{11} = 1$; $f_{12} = 0$; $f_{13} = 3$; $f_{21} = 1$; $f_{22} = 4$ $f_{23} = 0$ which yields a minimum cost of \$40.