MATH 465-001 Name________________________________

Mid-Term Exam Spring 2008

Directions. Work this test without the aid of any other person, and provide no aid to any other person. You may refer to your notes, textbook, or any other books you care to consult. Of course you may use a calculator or computer. Attach this cover sheet to your solutions and submit your completed exam no later than 9:30 am on Tuesday, April 1. Be sure to cite and credit any references you use.

Work these equally weighted problems in the spirit of the modeling approach used in your text. Pay particular attention to clearly communicating your assumptions, thoughts, and problem solving processes, and to justifying your procedures and conclusions or conjectures. Whenever possible, justify your answers by using mathematical concepts rather than "guess & test" procedures using a computer or calculator. This is an opportunity to show off what you know and have learned and how you can apply that knowledge.

Sign one of the two statements.

Statement: I worked this test in compliance with the above directions.

____ (Signature)

Statement: I am unable to sign the statement above due to the exception(s) listed below.

____ (Signature)

1. Exercises on pp. 23-24: #14 and #16

2. Exercises 2.1: #13

3. In what follows it is important to remember that we are simply considering a mathematical model. Its relevance to any particular real world population can only be verified by experiment.

   We revisit the case of fish in a pond. Recall we started with a few fish in a pond and considered a logistic model where the population increased rapidly at first and then leveled off around a fixed population size.

   Suppose the number of fish in the pond after n years, p_n, is modeled by the difference equation

$$p_0 = 300$$

$$(*) \qquad p_{n+1} - p_n = 0.001p_n(500 - p_n) \text{ for } n = 1, 2, , 3 \ldots$$

   This discrete model incorporates the implicit assumption that each year's incremental increase or decrease occurs at the end of each year, and is not distributed uniformly over the year. (This fact partially accounts for the fact that discrete and continuous models yield different predictions.)

   Suppose the pond's owner decides to remove 52 fish per year from the pond. (Again it is implicit in the model that the fish are all removed at once.) Will that rate of fishing deplete the pond's fish population?

   a. Revise equation (*) to model the situation where the pond's owner removes 52 fish from the pond each year, and determine if removing 52 fish per year will deplete the pond's fish population in the long run. If that rate will not deplete the population, will the population stabilize at any particular level?
   b. What is the maximum number of fish that can be removed each year without depleting the fish population in the pond?
   c. In your model for part (a), for what values of p_n is $p_{n+1} \geq p_n$?

4. Exercises 2.2 #9

5. Suppose the assumptions in our real model for lynx-hare interaction are modified as follows.

   Assumptions in the real model:

   - The lynx species is totally dependent on the hare species as its only food supply.
   - The hare species has a limited food supply; threats to its growth include both competition with other hares for the available food and the lynx predators.
   - The rate at which lynx encounter hares is jointly proportional to the sizes of the two populations.
   - The rate at which hares encounter each other in competition for food is proportional the square of the size of the hare population.

- A fixed proportion of encounters leads to the death of hares.
- The energy to support growth of the lynx population is proportional to deaths of prey.

a. Given those assumptions carefully formulate a discrete version of a mathematical model based on those assumptions.

b. Estimate values for the parameters in your model and produce graphs similar to those produced for the model developed in class. Identify any stable points you find.

7. Exercises 2.3 #20

8. Exercises 2.4 #8

9. Exercises 2.6: #10