Chicken Pecking Problem

Consider a chicken yard containing n chickens, for some $n \in Z$, which has a *well-defined pecking order*. That is, for any pair of distinct chickens c_i and c_j, either c_i pecks c_j or c_j pecks c_i, and not both.

We write $c_i > c_j$ to mean c_i pecks c_j.

Also, we write $c_i >> c_j$ to mean there is a chicken c_k such that $c_i > c_k$ and $c_k > c_j$.

A chicken c_d is said to be *dominant* iff for all other chickens c_k , $c_d > c_k$ or $c_d >> c_k$.

Does a chicken yard with a well-defined pecking order necessarily have a dominant chicken? (Prove or disprove.)

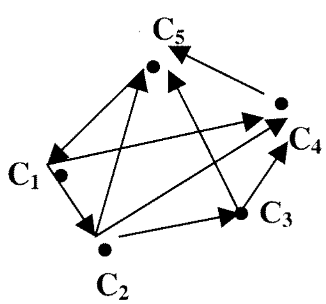
Example

Suppose $C = \{C_1, C_2, C_3, C_4, C_5\}$.

Suppose a pecking order is defined by

$C_1 > C_2$, $C_1 > C_4$
$C_2 > C_3$, $C_2 > C_4$, $C_2 > C_5$
$C_3 > C_1$, $C_3 > C_4$, $C_3 > C_5$
$C_4 > C_5$
$C_5 > C_1$

We can represent the pecking relation with a digraph *G* or a matrix *P*.



	C_1	C_2	C_3	C_4	C_5
C_1	0	1	0	1	0
C_2	0	0	1	1	1
C_3	1	0	0	1	1
C_4	0	0	0	0	1
C_5	1	0	0	0	0