

Chicken Pecking Example Revisited

$$P = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}_{5 \times 5}; \quad P_{ij} = \begin{cases} 1, & \text{if } c_i > c_j \\ 0, & \text{otherwise} \end{cases}$$

$$P^2 = PP = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

let $P^2 = (P_{ij}^{(2)})_{5 \times 5}$

Note, $P_{ij}^{(2)} = \sum_{k=1}^5 P_{ik} P_{kj}$ for $1 \leq i, j \leq 5$

Each product in the summation is either 0 or 1.

Moreover, $P_{ik} P_{kj} = \begin{cases} 1, & \text{if } c_i > c_k \text{ and } c_k > c_j \\ 0, & \text{otherwise.} \end{cases}$

So, $P_{ij}^{(2)} = 0$ if it is false that $c_i \gg c_j$

and $P_{ij}^{(2)} > 0$ if $c_i \gg c_j$.

- (1) What is the significance of the numerical value of $P_{ij}^{(2)}$?
- (2) What is the significance of the (i, j) -entry of $PP + P^2$?