

Discrete Math – Session 33

Example 1: Recall that a function $f: X \rightarrow Y$ (from X into Y) is onto S , where $S \subseteq Y$ iff $\forall y \in S \exists x \in X \text{ s.t. } f(x) = y$. Show that $f: \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $f(x) = 5 - 3x$ is onto \mathbb{Q} .

Example 2: Recall that a function $f: X \rightarrow Y$ is 1-1 iff $\forall x_1, x_2 \in X$ if $f(x_1) = f(x_2)$, then $x_1 = x_2$. Show that the function f defined in *Example 1* is 1-1.

Relations

Suppose A and B are sets. A **relation R from A to B** is a subset of $A \times B$. Given an ordered pair $(x, y) \in A \times B$, we say x is related to y by R , denoted by $x R y$, iff $(x, y) \in R$.

Example 3: Let $A = \{0, 1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$, suppose R is the relation from A to B defined by $R = \{(x, y) : x < y\}$. That is, $x R y$ iff $(x, y) \in R$. Or equivalently, $\forall x \in A \forall y \in B, x R y$ iff $x < y$.

List the ordered pairs in R .

Example 4: Consider the relation C_4 from \mathbb{Z} to \mathbb{Z} defined as follows: $\forall (m, n) \in \mathbb{Z} \times \mathbb{Z}, m C_4 n$ iff $4 \mid (m - n)$. Now, describe the set of all $m \in \mathbb{Z}$ such that $(m, 3) \in C_4$. That is, define by the listing method $\{m \in \mathbb{Z} : m C_4 3\}$.

Example 5: Consider the relation F from \mathbb{Q} to \mathbb{Q} defined by $(x, y) \in F$ iff $y = f(x)$ where f is the function of *Example 1*. Equivalently, $x F y$ iff $y = 5 - 3x$. Describe the set of ordered pairs in F .