

Relations – Properties & Examples

Suppose $C = C_1 \cup C_2$ where

$C_1 = \{\text{all the children of Amy and Bob}\} = \{\text{Carl ()}, \text{Dick ()}, \text{Ellen ()}, \text{Frank()}, \text{Ginger()}\}$

$C_2 = \{\text{all the children of Helen and Ivan}\} = \{\text{Jane()}, \text{Ken()}\}$

We define the relations B and S on $C \times C$ as follows:

$x B y$ iff $x = y$ and x is the brother of y

$x S y$ iff x and y have the same pair of parents

1. Place an “x” in the cell in row p , column q if $p B q$.

| B | Carl | Dick | Ellen | Frank | Ginger | Jane | Ken |
|--------|------|------|-------|-------|--------|------|-----|
| Carl | | | | | | | |
| Dick | | | | | | | |
| Ellen | | | | | | | |
| Frank | | | | | | | |
| Ginger | | | | | | | |
| Jane | | | | | | | |
| Ken | | | | | | | |

2. Place an “x” in the cell in row p , column q if $p S q$.

| S | Carl | Dick | Ellen | Frank | Ginger | Jane | Ken |
|--------|------|------|-------|-------|--------|------|-----|
| Carl | | | | | | | |
| Dick | | | | | | | |
| Ellen | | | | | | | |
| Frank | | | | | | | |
| Ginger | | | | | | | |
| Jane | | | | | | | |
| Ken | | | | | | | |

3. Determine the truth-value of each statement.

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|---|---|
| a. $\exists c_1 \forall C, c_1 B c_1$ | d. $\exists c_1 \forall C, c_1 S c_1$ |
| b. $\exists c_1, c_2 \forall C, \text{if } c_1 B c_2 \text{ then } c_2 B c_1$ | e. $\exists c_1, c_2 \forall C, \text{if } c_1 S c_2 \text{ then } c_2 S c_1$ |
| c. $\exists c_1, c_2, c_3 \forall C, \text{if } c_1 B c_2 \text{ and } c_2 B c_3, \text{ then } c_1 B c_3.$ | f. $\exists c_1, c_2, c_3 \forall C, \text{if } c_1 S c_2 \text{ and } c_2 S c_3, \text{ then } c_1 S c_3.$ |

4. a. List the elements in $\{q \in C : \text{Carl } S q\}$.

- List the elements in $\{q \in C : \text{Jane } S q\}$.