MATH 210 Discrete Math – Session 10

Negations of Quantified Statements

$\lim_{x \to a} f(x) = L$ iff $\forall \varepsilon > 0\ \exists \delta > 0 \ni \forall x \in D_f$ if $0 < |x - a| < \varepsilon$ then $|f(x) - L| < \varepsilon$

The negation of "$\forall \varepsilon > 0\ \exists \delta > 0 \ni \forall x \in D_f$ if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$" is the

statement "$\exists\ \varepsilon > 0 \ni \forall \delta > 0\ \exists x \in D_f \ni 0 < |x - a| < \delta$ and $|f(x) - L| \nless \varepsilon$."

Important Remarks in the Text

Note the next to last paragraph on p. 113. (The properties of R and the closure properties of Z)

Note the remarks about learning definitions at the top of page 114.

Note the remarks about constructive and nonconstructive proofs of existence on pp. 115-116.

Under the section titled "Proving Universal Statements" note the remarks about the proof technique called the ***method of generalizing from the generic to the particular*** on p. 117.

Note the remarks in the middle of p. 119 concerning use of the ***only if*** and ***if*** parts of a definition.

Carefully review the section titled "Directions for writing proofs of Universal Statements" on pp. 119-120. Consider the remarks about proofs written by different people on p. 121.

Be aware of the "Common Mistakes" discussed on pp. 120-121.

Prove: **If the sum of two integers is even, so is their difference.**
$\forall m \in Z\ \forall n \in Z$, if $m + n$ is even then $m - n$ is even.

Prove or disprove each of the following statements.

Claim: **$\forall r \in Q\ \forall s \in Q$, if $r < s$ then $\exists x \in Q \ni r < x < s$.**

Claim: **$\forall x \in R\ \forall y \in R$, if $x \notin Q$ and $y \notin Q$, then $xy \notin Q$.**