MATH 210 Discrete Math – Session 11

Prove: The product of any two odd integers is odd.

Proof: Suppose m and n are *[particular but arbitrary]* odd integers. *[By the **only if** (⇒) part of the definition of odd integer]* there exists integers i and j such that $m = 2i + 1$ and $n = 2j + 1$. *[We must show mn is odd.]* It follows that,

$$mn = (2i + 1)(2j + 1) = 4ij + 2i + 2j + 1 = 2(2ij + i + j) + 1.$$

[Since the set of integers is closed under multiplication and addition], $(2ij + i + j) = k$ for some integer k. So, *[by the **if** (⇐) part of the definition of odd number]* mn is an odd integer.

Hence, if m and n are odd integers, the mn is an odd integer.
That is, the product of any two odd integers is odd.

Prove or Disprove: For each integer n, $n^2 - n + 11$ is prime.

Prove or disprove: The sum of any four consecutive integers is even.

Definition: If n and d are integers and $d \neq 0$, then n is ***divisible by*** d iff $n = dk$ for some integer k. In this case we also say that

n ***is a multiple of*** d, or
d ***is a factor of n***, or
d ***is a divisor of n***, or
d ***divides*** n.

We denote "**d divides n**" by "**d | n**."

Prove or disprove: For all integers a, b, and c, if $a | b$ and $a | c$, then $a | (b + c)$.