MATH 210 Discrete Math – Sessions 15 & 16

Theorem 3.3.3 *Unique Factorization Theorem*
Given any integer n > 1 there exists a $k \in Z^+$, distinct prime numbers $p_1, p_2, \ldots, p_k$, and $e_1, e_2, \ldots, e_k \in Z^+$ such that

$$n = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_k^{e_k}$$

Any other factorization of n as the product of primes is identical to this except possibly for the order in which the primes are written.

Standard Factored Form

Example 3.3.11 **Suppose $m \in Z^+$ and $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot m = 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10$. Does $17 \mid m$?**

Theorem 3.4.1 *The Quotient-Remainder Theorem*
$\forall n \in Z \; \forall d \in Z^+ \; \exists! q, r \in Z \ni n = d \cdot q + r$ and $0 \le r < d$.

Given $n \ge 0$ and d as in the theorem above and the associated unique q and r,
n *div* d = q and n *mod* d = r

Theorem 3.4.3 **The square of any odd integer has the form $8m + 1$ for some integer m.**
(Proof by cases)

Theorem 3.6.2 **The sum of any rational number and any irrational number is irrational.**
(Proof by contradiction)

Proposition 3.6.3 **$\forall n \in Z$ if n^2 is even then n is even.**
(Proof by contaposition.)

Theorem 3.7.1 **$\sqrt{2}$ is irrational.**

Theorem 3.7.4 **The set of primes is infinite.**