MATH 210 Discrete Mathematics – Session 18

Wrap-up remarks on summation and product notation. (See Session 17.)

1. Compute the sum or product:

a. $\sum_{n=1}^{5}\left(\frac{1}{n}-\frac{1}{n-1}\right)$

b. $\prod_{i=2}^{4}\frac{(i-2)(i-1)}{i(i+1)}$

Principle of Mathematical Induction

Let P(n) be a predicate that is defined for integers n, and let a be a fixed integer. Suppose the following two statements are true:

1. **P(a) is true.**
2. **For all integers k $\geq$ a, if P(k) is true then P(k + 1) is true.**

Then the statement

For all integers n $\geq$ a, P(n)

is true.

Example: **For any positive integer n, 1 + 3 + … + (2n – 1) = n^2**

Proof using the Principle of Mathematical Induction:

In this case we are trying to establish that $\forall n \in Z^+$ P(n) where P(n) is given by P(n): 1 + 3 + … + (2n-1) = n^2.

Basis Step: **P(1) is true because $(2\cdot 1 - 1) = 1 = 1^2$.**

Inductive Step: **Assume that P(k) is true for some $k \in Z^+$.**
[*This is our inductive hypothesis.*] [*We must now show that P(k + 1) is true.*]

That is, we assume that

1 + 3 + … + (2k – 1) = k^2 for some [*arbitrary but fixed*] positive integer k.

Adding [2(k + 1) – 1] to both sides of the above equation, it follows that

1 + 3 + … + (2k – 1) + [2(k + 1) – 1] = k^2 + [2(k + 1) – 1]

Applying algebra to the right hand side we obtain

1 + 3 + … + (2k – 1) + [2(k + 1) – 1] = $k^2 + 2k + 1$
= $(k + 1)^2$

So, P(k + 1), or 1 + 3 + … + (2k – 1) + [2(k + 1) – 1] = $(k + 1)^2$, is true.
We have shown that P(k) for some $k \in Z^+$ implies P(k + 1).
We have now established that P(1) is true and that for any $k \in Z^+$ P(k) $\rightarrow$ P(k+1).

Consequently, by the Principle of Mathematical Induction,
For any positive integer n, 1 + 3 + … + (2n – 1) = n^2.