MATH 210 Discrete Mathematics - Sessions #24 and #25

Overview of Set Theory

Undefined Terms:

objects – set, element
relation – set membership

Definitions:

special sets: universal set, empty set
special objects: ordered pair, ordered n-tuple
special relation: equality of ordered n-tuples
relations – subset, equality of sets
binary operations – intersection, union, difference or relative complement, Cartesian product
unary operation – complement

Theorems:

For all A, B, C subsets of a universal set *U*,
Some Subset Relations

1.	$A \cap B \subseteq A$ and $A \cap B \subseteq B$	(Inclusion of Intersection)
2.	$A \subseteq A \cup B$ and $B \subseteq A \cup B$	(Inclusion in Union)
3.	If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$	(Transitivity of Subset Relation)

Set Identities

4.	$A \cap B = B \cap C$ and $A \cup B = B \cup A$	(Commutative Laws)
5.	$(A \cap B) \cap C = A \cap (B \cap C)$ and $(A \cup B) \cup C = A \cup (B \cup C)$	(Associative Laws)
6.	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	(Distributive Laws)
7.	$A \cap U = A$	(Identity for $\cap$)
8.	$(A^c)^c = A$	(Double Complement Law)
9.	$A \cap A = A$ and $A \cup A = A$	(Idempotent Laws)
10.	$(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$	(De Morgan's Laws)
11.	$A \cup U = U$	(Universal bound for $\cup$)
12.	$A \cup (A \cap B) = A$ and $A \cap (A \cup B) = A$	(Absorption Laws)
13.	$A - B = A \cap B^c$	(Alternative Representation)

Some More Theorems

14. For any sets A and B, if A ⊆ B then
    (a) A ∩ B = A and (b) A ∪ B = B.
15. If A is any set, N ⊆ A.
16. For any subset A of a universal set *U*,
    a. A ∪ N = A (Identity for ∪)
    b. A ∩ A^c = N and A ∪ A^c = *U*
    c. A ∩ N = N (Universal bound for ∩)
17. U^c = N and N^c = *U*
18. For any sets A and B, (A – B) and B are disjoint.
19. For all sets A and B, if A ⊆ B then P (A) ⊆ P (B).
20. For all in tegers n $\geq$ 0 and sets X, if X has exactly n elements, then P(X) has exactly 2^n elements.

Boolean Algebras

A Boolean algebra (S, =, +, *) is a set S together with an equality relation (=) and two binary operations (+ and *) such that the following properties hold:

For all a, b, c in S,

0. S is closed under both + and *
1. a + b = b + a and a * b = b * a
2. (a + b) + c = a + (b + c) and (a*b)*c = a*(b*c)
3. a +(b*c) = (a+b)*(a+c) and a*(b+c) = (a*b) + (a*c)
4. There exists distinct elements i and e in S such that
   a + i = a and a * e = a
5. For each a is S there exists an element a^{-1} such that a * a^{-1} = e and a + a^{-1} = i.

Theorem. Suppose S is *the set of all subsets* of of a nonempty set U, that is S = P(U), then (S, = , c, 1) is a Boolean algebra.