Equivalence Relations

Let A be a nonempty set and R a relation on A (a subset of $A \times A$), is an *equivalence relation* on A iff R is reflexive, symmetric, and transitive.

Example: For some $d \in Z^+$ we define the relation C_d on Z as follows:

> **$\forall m, n \in Z$ $m\ C_d\ n$, denoted by $m \equiv n \pmod{d}$, iff $d \mid (m - n)$. We read "$m \equiv n \pmod{d}$" as "m is congruent to n modulo d."**

Is congruence modulo d an equivalence relation?

Consider the relation congruence modulo 5. List all the elements in each of the following sets:

$\{x \in Z: x \equiv 0 \pmod{5}\}$:
$\{x \in Z: x \equiv 1 \pmod{5}\}$:
$\{x \in Z: x \equiv 2 \pmod{5}\}$:
$\{x \in Z: x \equiv 3 \pmod{5}\}$:
$\{x \in Z: x \equiv 4 \pmod{5}\}$:
$\{x \in Z: x \equiv 5 \pmod{5}\}$:

Note that the sets above partition Z.

We denote $\{x \in Z: x \equiv a \pmod{5}$ where $a \in Z\}$ by [a].

For $a, b \in Z$ how can we determine whether or not [a] = [b]?

Some Counting Exercises

1. How many functions are there from {1, 2, 3} into {1, 2, 3, 4}?

2. How many functions are there from {1, 2, 3, 4} into {1, 2, 3}?

3. How many 1-1 functions are there from {1, 2, 3} into {1, 2, 3, 4}?

4. How many 1-1 functions are there from {1, 2, 3, 4} into {1, 2, 3}?

5. How many functions are there from {1, 2, 3} onto {1, 2, 3, 4}?

6. How many functions are there from {1, 2, 3, 4} onto {1, 2, 3}?

7. How many 1-1 functions are there {1, 2, 3} onto {1, 2, 3}?

8. How many 3-digit numerals can we write using just the symbols 1, 2, 3?

9. How many 3-digit numerals can we write using just the symbols 1, 2, 3 if we can use each symbol exactly once?