MATH 210 Discrete Mathematics – Session 9

On what basis do we claim the empty set is a subset of each set?

Consider the *conditional* statement form
(*) $\forall x \in D$, if P(x) then Q(x)

Its *contrapositive* is the statement ___________________________________.

Its *converse* is the statement ___________________________________.

Its *inverse* is the statement ___________________________________.

Statement (*) means "$\forall x \in D$, __________ is a *sufficient condition* for _________."

Statement (*) means "$\forall x \in D$, __________ is a *necessary condition* for ________."

Statement (*) means "$\forall x \in D$, __________ *only if* _________.

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Prove: **The sum of two even numbers is even.**
$\forall m \in Z\ \forall n \in Z$, if m is even and n is even then m + n is even.

Proof:	**(1)**	**m and n are arbitrary even integers.**	**Premise**
	(2)	**$\forall j \in Z$, j is even iff $\exists k \in Z$ such that j = 2k**	**Definition**
	(3)	**m is an even integer**	____________
	(4)	**n is an even integer**	____________
	(5)	**m = 2r for some r $\in$ Z**	____________
	(6)	**n = 2s for some s $\in$ Z**	____________
	(7)	**m + n = m + n**	____________
	(8)	**m + n = 2r + 2s**	____________
	(9)	**2r + 2s = 2(r + s)**	____________
	(10)	**m + n = 2(r + s)**	____________
	(11)	**(r + s) $\in$ Z**	____________
	(12) $\therefore$	**m + n is even**	____________
	(13) $\therefore$	**if m is even and n is even then m + n is even**	____________

Consequently,
$\forall m \in Z\ \forall n \in Z$, if m is even and n is even then m + n is even.

Alternative Paragraph Proof:

Suppose m and n are arbitrary even integers. By the definition of even integer, there exists integers r and s such that m = 2r and n = 2s. Now, m + n = 2r + 2s = 2(r + s). But (r + s) is an integer; so m + n is even. Hence, if m and n are arbitrary even integers, then m + n is aslo an even iteger. Consequently, $\forall m \in Z\ \forall n \in Z$, if m is even and n is even then m + n is even.