Chapter 5 Section 2

7) Prove by mathematical induction that for any integer $n \geq 1$ and all sets $A_1, A_2, \ldots A_n$ and B,
$(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_n - B) = (A_1 \cap A_2 \cap \ldots \cap A_n) - B$.
Assume that if $n \geq 3$ and $C_1, C_2, \ldots C_n$ are any sets, $C_1 \cap C_2 \cap \ldots \cap C_n$ is defined to be
$(C_1 \cap C_2 \cap \ldots \cap C_{n-1}) \cap C_n$.

Proof:

1) Basis step

The formula holds for n = 1.

For n = 1 the formula is $A_1 - B = A_1 - B$ which is clearly true.

2) Inductive step

If the formula holds for n = k then it hold for n = k + 1.

Let k be an integer with $k \geq 1$, and suppose the formula holds for n = k. I must show that the formula holds for n = k + 1; that is for any sets $A_1, A_2, \ldots A_{k+1}$, and B.

$$(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_{k+1} - B) = (A_1 \cap A_2 \cap \ldots \cap A_{k+1}) - B$$

But

$$(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_{k+1} - B) = [(A_1 - B) \cap (A_2 - B) \cap \ldots (A_k - B)] \cap (A_{k+1} - B)$$

~~*By assumption*~~

$$= [(A_1 \cap A_2 \cap \ldots \cap A_k) - B] \cap (A_{k+1} - B)$$

By the inductive hypothesis

$$= [(A_1 \cap A_2 \cap \ldots \cap A_k) \cap A_{k+1}] - B$$

By example 5.2.5 ✓ uses unions exercise 5.2/11

$$= (A_1 \cap A_2 \cap \ldots \cap A_k \cap A_{k+1}) - B$$

By assumption

Hence, the statement, "For any integer $n \geq 1$ and all sets $A_1, A_2, \ldots A_n$ and B, $(A_1 - B) \cap (A_2 - B) \cap \ldots \cap (A_n - B) = (A_1 \cap A_2 \cap \ldots \cap A_n) - B$," is true by steps 1 and 2 and the Principle of Mathematical Induction.