9) Prove that for all sets A and B, $(A - B) \cup (A \cap B) = A$

Proof:

Suppose A and B are arbitrarily chosen sets.

Show $(A - B) \cup (A \cap B) \subseteq A$:

Suppose $x \in (A - B) \cup (A \cap B)$. By definition of union, $x \in (A - B)$ or $x \in (A \cap B)$

Case 1 $x \in (A - B)$

Then by definition of set difference $x \in A$ and $x \notin B$.

In particular, $x \in A$

Case 2 $x \in (A \cap B)$

Then by definition of intersection $x \in A$ and $x \in B$.

In particular, $x \in A$.

Hence, in either case $x \in A$, and so by definition of subset $(A - B) \cup (A \cap B) \subseteq A$

Show $A \subseteq (A - B) \cup (A \cap B)$.

Suppose $x \in A$. Either $x \in B$ or $x \notin B$.

Case 1: $x \in B$

Then since $x \in A$ also, by definition of intersection $x \in (A \cap B)$,

And so by the inclusion in union property, $x \in (A - B) \cup (A \cap B)$.

Case 2: $x \notin B$

Then since $x \in A$ also, by definition of set difference $x \in (A - B)$,

and so by definition of subset, $A \subseteq (A - B) \cup (A \cap B)$.

Hence, in either case $x \in (A - B) \cup (A \cap B)$, and so by definition of

subset, $A \subseteq (A - B) \cup (A \cap B)$.

Since both set containments have been proved, $(A - B) \cup (A \cap B) = A$ by definition of set equality.