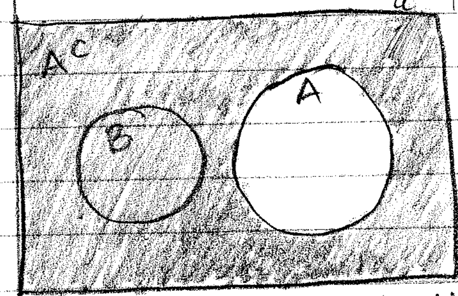
11. For all sets A and B, if $B \subseteq A^c$ then $A \cap B = \emptyset$

Illustration

True



Proof: by contradiction

Let A and B be sets such that $B \subseteq A^c$. We must show that $A \cap B = \emptyset$. Suppose $A \cap B \neq \emptyset$; that is suppose there were an element x such that $x \in A \cap B$. Then $x \in A$ and $x \in B$, by definition of intersection. But $B \subseteq A^c$ by hypothesis. So since $x \in B$, $x \in A^c$ by definition of subset. So $x \notin A$ by def of complement. Thus $x \in A$ and also $x \notin A$, which is a contradiction. Hence the that $A \cap B \neq \emptyset$ is false, and so $A \cap B = \emptyset$. Therefore if $B \subseteq A^c$ then $A \cap B = \emptyset$.

Since this is a proof by contradiction, I need to state my contradiction then ~~pro~~ follow through w/ logical steps till I come to a contradiction. In this case my contradiction was $x \in A$ and $x \notin A$.