

17. Prove the following generalization employing the method of direct proof. Employ excellent style.

Prove: The product of any two odd integers is odd.

Suppose m and n are arbitrary, but particular, odd integers. By the definition of odd, there exists some integer k such that $m = 2k + 1$. There also exists some integer r such that $n = 2r + 1$. We are to show that the product of any two odd integers is odd. We can say that $m \cdot n = (2k+1)(2r+1)$ by substitution. So,

$$\begin{aligned} m \cdot n &= (2k+1)(2r+1) \\ &= 4kr + 2k + 2r + 1 \\ &= 2(2kr + k + r) + 1 \end{aligned}$$

We know that there exists some integer s , such that $s = 2kr + k + r$ because integers are closed under multiplication and addition. So, $m \cdot n = 2s + 1$, ~~and~~ By the definition of odd, $m \cdot n$ is odd.

Hence the product of any two odd integers is odd.