(20) The sum of any even and any odd integer is odd.

Proof

Suppose m and n are integers such that m is even and n is odd. Then exists integers i and j such that $m = 2i$ and $n = 2j+1$. It follows that

$$m+n = 2i+2j+1$$
$$= 2(i+j)+1$$

since the set of integers is closed under multiplication and addition, $i+j = n$ for some integer n.

Therefore, $m+n = 2n+1$. So by the if ($\Leftarrow$) part of the definition of odd number, $m+n$ is an odd integer

Hence, if m is even and n is odd, then $m+n$ is odd. That is, the sum of any even and any odd integer is odd.

Good

Once again, I wasn't really sure where I was going when I started this proof. I just followed through the steps of a proof and it isn't until I get through the algebra that it all comes together.