4.2

(10) To Prove:

$1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$, for all integers $n \geq 1$.

Proof by PMI:

Basis (1) Let $n=1$: $(1)^3 = 1$ and $\left[\frac{(1)[(1)+1]}{2}\right]^2 = \left[\frac{2}{2}\right]^2 = 1^2 = 1$.

So, we have shown that the formula works for $n=1$.

Inductive Hypothesis (2) Suppose the formula is true for $n=k$.

That is, $1^3 + 2^3 + \ldots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$

Now we must show that the formula holds true for $n=k+1$.

$$(1^3 + 2^3 + \ldots + k^3) + (k+1)^3 = \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$$
$$= (k+1)^2\left[\left(\frac{k}{2}\right)^2 + (k+1)\right]$$
$$= (k+1)^2\left[\frac{k^2}{4} + \frac{4(k+1)}{4}\right] \quad \text{(because } \tfrac{4}{4} = 1\text{)}$$
$$= (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$$
$$= (k+1)^2\left[\frac{(k+2)^2}{4}\right]$$
$$= \left[\frac{(k+1)(k+2)}{2}\right]^2 = \left[\frac{(k+1)[(k+1)+1]}{2}\right]^2$$

good

So, we have shown that if the formula holds true for $n=k$ then ~~and~~ it holds for $n=k+1$.

Therefore, by (1), (2), and PMI, the formula is true for all integers $n \geq 1$.

(Having 4.2/#7 as an example for how to word things, and by your help, I think I did this correctly. Practicing it more is also helpful!)