Muhammad Mustafa

Exercise Set 4.3

(2) Experiment with computing values of $(1+\frac{1}{1})(1+\frac{1}{2})(1+\frac{1}{3})\cdots$ $\cdots(1+\frac{1}{n})$ for small values of n to conjecture a formula for this product for general n. Prove your conjecture by mathematical induction.

Proof:

In this case, the formula for $P(n)$ would be

$P(n)$: $\prod_{i=1}^{n}\left(1+\frac{1}{i}\right) = n+1$ for all integers $n \geq 1$.

(1) Basis Step:

The formula holds for $n=1$ because,

$P(1)$: $\prod_{i=1}^{1}\left(1+\frac{1}{i}\right) = 1+\frac{1}{1} = \frac{2}{1} = 2 = (1+1)$

(2) Inductive Step:

If the formula is true for $n=k$, it is true for $n=k+1$

Suppose $\prod_{i=1}^{k}\left(1+\frac{1}{i}\right) = k+1$ for some integer $k \geq 1$ — Inductive hypothesis

We must show that

$\prod_{i=1}^{k+1}\left(1+\frac{1}{i}\right) = (k+1)+1 = k+2$ to be true.

Therefore

$\prod_{i=1}^{k+1}\left(1+\frac{1}{i}\right) = \prod_{i=1}^{k}\left(1+\frac{1}{i}\right)\cdot\left(1+\frac{1}{k+1}\right)$ — by making next to last terms explicit.

$= k+1 \cdot \left(\frac{(k+1)+1}{k+1}\right)$ — By laws of algebra and substitution from Inductive hypothesis.

$= \cancel{k+1} \cdot \frac{k+2}{\cancel{k+1}}$

$= k+2.$

good

Hence, $\prod_{i=1}^{n}\left(1+\frac{1}{i}\right) = n+1$ is proved for all integers $n \geq 1$. Therefore the conjecture is proved by mathematical Induction. [This is what we needed to show