4) Proposistion: $2^n < (n+2)!$ for all integers $n \geq 0$

Proof by Mathematical Induction

(1) The proposition is true for $n=0$: To show that the proposition is true for $n=0$, we must show that $2^0 < (0+2)!$. But $2^0 = 1$, $(0+2)! = 2$, and $1<2$. Hence the proposition is true for $n=0$.

(2) If the proposition is true for $n=k$, then it is true for $n=k+1$: Suppose $2^k < (k+2)!$ for some integer k such that $k \geq 0$. [This is the inductive hypothesis.] We must show that $2^{k+1} < ((k+1)+2)!$, or, equivalently, $2^{k+1} < (k+3)!$ But,

$$2^{k+1} = (2^k)(2) \text{ and } (k+3)! = (k+2)!(k+3)$$

So we must show that $(2^k)(2) < (k+2)!(k+3)$. By our inductive hypothesis, $2^k < (k+2)!$. Because $k \geq 0$, $k+3 \geq 3$. Since $2<3$, when we multiply (2^k) by 2 and $(k+2)!$ by $k+3$, the inequality must remain true. (We are multiplying the smaller quantity by a smaller number than that of the larger quantity) So,

$$2^{k+1} < (k+3)!$$

Thus, if the proposition is true for $n=k$, then it is true for $n=k+1$.

By (1), (2), and PMI, $2^n < (n+2)!$ for all integers $n \geq 0$