Sample Solutions

State what you hope to establish.
Follow the prescribed format for the type of argument you will use.
Proceed with the argument providing appropriate support for each step.
Clearly state your conclusion.

3.4/#25
Prove: The square of any integer has the form 4k or 4k + 1 for some $k \in Z$.

Proof:
Suppose n is an arbitrary integer. By the parity property, n is either even or odd. We will consider each case.
Case 1: Suppose n is even. There must exist $j \in Z$ such that $n = 2j$. It follows that $n^2 = 4j^2$. But $j^2 = k$ for some $k \in Z$ and $n^2 = 4k$.
Case 2: Suppose n is odd. There must exist $j \in Z$ such that $n = 2j + 1$. It follows that $n^2 = 4j^2 + 4j + 1 = 4(j^2 + j) + 1$. But $j^2 + j = k$ for some $k \in Z$ and $n^2 = 4k + 1$.
So, in each case n^2 has the form 4k or 4k + 1 for some $k \in Z$.
Consequently, the square of any integer has the form 4k or 4k + 1 for some $k \in Z$.

3.6/#11
Prove: For each integer n and prime p, if $p|n^2$, then $p|n$.

Proof by Contradiction:
Suppose n is an arbitrary integer and suppose p is a prime number such that $p|n^2$. We hope to show that $p|n$.
Suppose that $p\nmid n$. So, p cannot be a prime factor of n. By 3.3/#34 n and n^2 have the same prime factors. Hence, $p\nmid n^2$. We have a contradiction of our premise that $p|n^2$.
Therefore, it cannot be the case that $p\nmid n$. So, $p|n$.
Hence, if $p|n^2$, then $p|n$.
Consequently,for each integer n and prime p, if $p|n^2$, then $p|n$.

Proof by Contraposition:
We will prove the contrapositive of the statement "For each integer n and prime p, if $p|n^2$, then $p|n$." Subsequently, we will invoke the equivalence of the contrapositive and the conditional.
Suppose that n is an arbitrary integer and p is a prime such that $p\nmid n$. p cannot be in the prime factorization of n. So, by 3.3/#34 p cannot be in the prime factorization of n^2. So, $p\nmid n^2$.
Hence, if $p\nmid n$, then $p\nmid n^2$.
Therefore, for each integer n and prime p, if $p\nmid n$, then $p\nmid n^2$.
Consequently, applying the equivalence of the contrapositive, for each integer n and prime p, if $p|n^2$, then $p|n$.