Prove: $\sqrt{5}$ is irrational.

Proof by Contradiction:

Suppose $\sqrt{5}$ is rational. There then exists a, b 0 Z^+ such that b > 0 and

(*) $\sqrt{5} = \frac{a}{b}$.

Squaring both sides of (*) gives $5 = \frac{a^2}{b^2}$ or equivalently,

(**) $5b^2 = a^2$.

Since 5 is a prime factor of a^2, by 3.3/#34 the exponent of 5 in the prime factorization of a^2 must be even. But, by 3.3/#34 again and properties of exponents, then 5 must appear to an odd power in the prime factorization of $5b^2$. This contradicts the Unique Factorization Theorem – the powers of 5 must be the same number on both sides of (**).

Hence our assumption that $\sqrt{5}$ is rational must be false, and therefore $\sqrt{5}$ is irrational.