

Part I. Short Answer (2 points each).

1. Write the contrapositive of "If  $n$  is divisible by 6, then  $n$  is divisible by 2."
2. Write the converse of "If  $n$  is divisible by 6, then  $n$  is divisible by 2."
3. Write the negation of "If  $n$  is divisible by 6, then  $n$  is divisible by 2" in the form of a conjunction.
4. Rewrite the statement "A late penalty will be charged, unless payment is received by the first of the month" in if-then form.

Some of the arguments in 5-8 are valid while others are not valid. If an argument is valid, identify the rule of inference that guarantees its validity. Otherwise state whether the converse or inverse error is made.

5. Tom is tall or Mary is young.  
Mary is not young.  
 $\neg$  Tom is tall.
6. If Tom is tall, then Mary is young.  
Tom is not tall.  
 $\neg$  Mary is not young.
7. If Tom is tall, then Mary is young.  
Mary is not young.  
 $\neg$  Tom is not tall.
8. Tom is tall or Mary is young  
If Tom is tall, then Sue is twenty-one.  
If Mary is young, then Sue is twenty-one.  
 $\neg$  Sue is twenty-one.

Some of the statements in 9-12 are true while others are false. If a statement is true, say so. Otherwise say so and provide a counterexample.

9.  $\exists x \in \mathbb{R}, x \neq 1/x$ .
10.  $\exists n \in \mathbb{Z}^+$  if  $n$  is prime and  $n > 5$ , then  $n$  is odd.

11.  $\forall n, m \in \mathbb{Z}$  if  $n - m$  is even then  $n^4 - m^4$  is even.

12.  $\forall n \in \mathbb{Z}^+ \exists a, b \in \mathbb{Z}^+, n = (a/b)^2$ .

Write negations for the statements in 13 and 14.

13.  $\exists n \in \mathbb{Z}^+ \forall m \in \mathbb{Z}^+, n \neq m$ .

14.  $\forall n \in \mathbb{Z}^+$  if  $n$  is prime, then  $n$  is odd.

15. Show how to use truth tables to establish the equivalence of the statement forms  $\sim(p \vee q)$  and  $\sim p \wedge \sim q$ .

Part II. Exercises (6 points each).

16. A set of premises and a conclusion are given below. Use our valid argument forms to deduce the conclusion from the premises, giving a reason for each step. Use only as many numbered steps as you need.

(1)  $\sim p \wedge q$  premise

(2)  $\sim q$  premise

(3)  $(\sim p \vee \sim q) \rightarrow r$  premise

(4) \_\_\_\_\_

(5) \_\_\_\_\_

(6) \_\_\_\_\_

$\sim r$  \_\_\_\_\_

17. Stated on next page.

17. Prove the following generalization employing the method of direct proof. Employ excellent style.

Prove: The product of any two odd integers is odd.