Name
Spring 2003

## Part I. Short Exercises \& Definitions (4 points each).

1. Consider the Venn diagram below. Shade in the region corrsponding to $A^{c} C B^{c}$.

2. Use set identities (in Theorem 5.2.2) to show that for all subsets $\mathrm{A}, \mathrm{B}$, and C of a universal set $U$, A $1(\mathrm{~B}-\mathrm{C})=(\mathrm{A}-\mathrm{C}) 1 \mathrm{~B}$.
3. Suppose $A=\{1,2,3\}$. Display (A) by listing its elements.
4. Consider the graph $G$ shown below.

$v_{4}$
a. Find $\operatorname{deg}\left(v_{2}\right)$ :
b. Find total degree of G:
c. Does G have an Euler curcuit? Justify your answer.
5. Is it possible to taker a walk around the city whose map is shown below, starting and ending at the same point and crossing each bridge exactly once? If not, why not? If so, how can it be done?
6. Suppose the matrix A below is the adjacency matrix for a directed graph. Sketch a picture of the directed graph with adjaceny matrix A, and show how to use $\mathrm{A}^{2}$ to find the number of paths of length 2 from $v_{1}$ to $v_{4}$.

7. Suppose $f: \mathrm{X} \quad$ Y. Define what we mean when we say $f$ is one-to-one.
8. Suppose $f$ : $\mathrm{X} \quad \mathrm{Y}$. Define what we mean when we say $f$ is onto Y.

Part II. Proofs. (6 points each). Use correct format and style in writing up each proof. Be sure you clearly state your assumptions and what you have proved.
9. Prove directly from the definitions of set operations: For all sets $A, B$, and $C$, if A f B then A 1 Cf B 1 C.
10. Prove or disprove directly from the definition of one-to-one:
$f:-\{3\} \quad$ defined by $f(x)=\frac{x}{x-3}$ is one-to-one.
11. Prove or disprove directly from the definition of onto:

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f:-\{3\} \quad \cdot \text { defined by } f(\mathrm{x})=\frac{x}{x-3} \text { is onto } \cdot
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