We have considered three major types of functions. Let's review some of what we have observed.

Example 1. Consider the relationship illustrated in the following table and graph.

| $\Delta \mathrm{n}$ | n | $\mathrm{f}(\mathrm{n})$ | $\Delta \mathrm{f}(\mathrm{n})$ | $\Delta \mathrm{f}(\mathrm{n}) / \Delta \mathrm{n}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 4 |  |  |
| 1 | 1 | 10 | 6 |  |
| 1 | 2 | 16 | 6 |  |
| 1 | 3 | 22 | 6 |  |
| 1 | 4 | 28 | 6 |  |
| 1 | 5 | 34 | 6 |  |
| 1 | 6 | 40 | 6 |  |

Define the relationship recursively with a
 difference equation.

Define the relationship explicitly with a functional equation.

Example 2. Consider the relationship illustrated in the next table and graph.

| $\Delta \mathrm{n}$ | n | $\mathrm{h}(\mathrm{n})$ | $\Delta \mathrm{h}(\mathrm{n})$ | $\mathrm{h}(\mathrm{n}) / \mathrm{h}(\mathrm{n}-1)$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 5 |  |  |
| 1 | 1 | 10 | 5 |  |
| 1 | 2 | 20 | 10 |  |
| 1 | 3 | 40 | 20 |  |
| 1 | 4 | 80 | 40 |  |
| 1 | 5 | 160 | 80 |  |
| 1 | 6 | 320 | 160 |  |

Define the relationship recursively with a difference equation.


Define the relationship explicitly with a functional equation.

Example 3. Consider the relationship defined by the following table. Complete the table and graph.

| $\Delta \mathrm{n}$ | n | $\mathrm{k}(\mathrm{n})$ | $\Delta \mathrm{k}(\mathrm{n})$ | $\Delta \Delta \mathrm{k}(\mathrm{n})$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 1 | 1 | 8 |  |  |
| 1 | 2 | 17 |  |  |
| 1 | 3 | 30 |  |  |
| 1 | 4 | 47 |  |  |
| 1 | 5 | 68 |  |  |
| 1 |  |  |  |  |

Define the relationship recursively with a difference equation.

Define the relationship explicitly with a functional equation.


Example 4. $\mathbf{y}=\mathbf{m x}+\mathrm{b}$

| $\mathbf{x}$ | $\mathbf{y}$ | $\Delta \mathbf{y}$ |
| :---: | :---: | :---: |
| $\mathbf{0}$ |  |  |
| $\mathbf{1}$ |  |  |
| $\mathbf{2}$ |  |  |
| $\mathbf{3}$ |  |  |
| $\mathbf{4}$ |  |  |

Example 5. $\quad \mathrm{y}=\mathrm{A}(\mathrm{r})^{\mathrm{x}}$

| $\mathbf{x}$ | $\mathbf{y}$ | ratio |
| :---: | :---: | :---: |
| $\mathbf{0}$ |  |  |
| $\mathbf{1}$ |  |  |
| $\mathbf{2}$ |  |  |
| $\mathbf{3}$ |  |  |
| $\mathbf{4}$ |  |  |

Example 6. $y=a x^{2}+b x+c$

| $\mathbf{x}$ | $\mathbf{y}$ | $\Delta \mathbf{y}$ | $\Delta \Delta \mathbf{y}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{0}$ |  |  |  |
| $\mathbf{1}$ |  |  |  |
| $\mathbf{2}$ |  |  |  |
| $\mathbf{3}$ |  |  |  |
| $\mathbf{4}$ |  |  |  |

