Difference Equations & Functional Equations

1. **Consider the sequence: 3, 8, 13, 18, …**

   a. **The first term is _____. We will denote this by $a_1 = 3$.**
   b. **The second term is ____. We will denote this by a_2 = ____.**
   c. **The third term is _____. We will denote this by a_3 = ____.**
   d. **The fourth term is the third term plus ____. We write $a_4 = a_3$ + ____.**
   So, a_4 = ____.
   e. **The fifth term is the fourth term plus ____. We write $a_5 = a_4$ + ____.**
   So, a_5 = ____.
   f. **$a_6 = a_5$ + ____ = ____.**
   g. **The next term in the sequence is always the previous term plus ____.**
   k. **The n^{th} term is the $(n-1)^{st}$ term plus ____.**
   l. **$a_n = a_{n-1}$ + ____.** ***(This is called a difference equation.)***

This sequence can be defined by recursion via a difference equation:

$a_1 = 3$ **and,**
$a_n = a_{n-1} + 5$ **for** $n \geq 1$.

This sequence can also be defined explicitly by the functional equation

$a_n = 5n - 2.$

2. **Consider the sequence in the following table.**

n	1	2	3	4	5	6	7
b_n	5	9	13	17	21		

a. Complete the table.

b. Is this an arithmetic sequence? Why or why not?

c. Define the sequence recursively using a difference equation.

d. Define the sequence explicitly using a functional equation.

3. **Consider the sequence in the following table. Here we start with n = 0.**

n	0	1	2	3	4	5	6
c_n	32	40	50	62.5			

1st differences:

n	0	1	2	3	4	5	6
c_n	32	40	50	62.5			

Ratio $\frac{c_n}{c_{n-1}}$ **:**

a. Complete the table.

b. Is this an arithmetic sequence? (Why or why not?)

c. A sequence in which each successive term is obtained by multiplying the previous term by a fixed number is called a *geometric sequence*. The fixed number is called the *common ratio*. Is the sequence of this example a geometric sequence? If so what is its common ratio?

d. Define this sequence recursively using a difference equation.

e. Define this sequence explicitly using a functional equation.

4. **Consider the sequence in the following table.**

n	0	1	2	3	4	5	6	7	8
d_n	4	9	20	37	60	89			

1st differences:

2nd differences:

a. Complete the table.

b. Define the sequence recursively using a difference equation.

c. Later we will define the sequence explicitly using a functional equation.