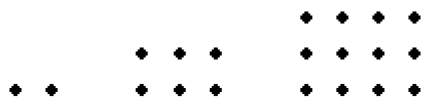
Figurate Numbers

Figurate numbers **can be represented by dots arranged in the shape of certain geometric figures. For example, the 1st three** ***rectangular numbers*** **are shown below.**

1. Consider the sequence of ***rectangular numbers*** **whose 1st three terms are shown below.**

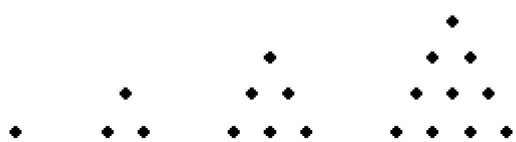


2 6 12

Find the next three rectangular numbers by drawing the corresponding arrays. Let R_n denote the n^{th} rectangular number. Complete the table below and find R_{10} and R_{20}. Try to find a rule for finding R_n for any value of n.

n	1	2	3	4	5	6	7	8	9	10	11
R_n											

2. The 1st four ***triangular numbers*** **are shown below.**

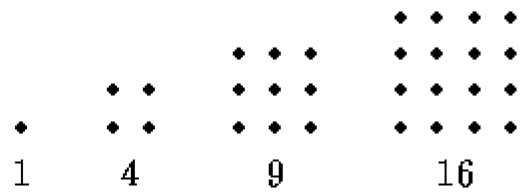


1 3 6 10

Determine the next three triangular numbers by drawing the corresponding trianglular arrays. Let T_n denote the n^{th} triangular number. Complete the table below and find T_{10} and T_{20}. Try to find a rule for finding T_n for any value of n.

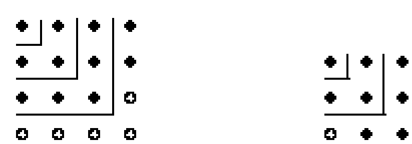
n	1	2	3	4	5	6	7	8	9	10	11
T_n											

3. The 1st four *square numbers* are shown below.



Suppose we denote the n^{th} square number by S_n. Write a rule for determining S_n for any n.

To the ancient Greeks, the square root of a number in this sequence was the number of dots along one side of the square that represents the number. For non-square natural numbers, they used a clever technique to estimate the square roots. This technique is illustrated below.



$\sqrt{11} \approx 3\frac{2}{7}$ $\sqrt{8} \approx 2\frac{4}{5}$

Use this technique to arrive at the approximation $[\sqrt{22}] \approx 4^6/_9$.