Ch4/#25

We will take note of the times the two full-time employees work and then determine the number of part-time employees we need each hour.

Time	Number of Part-Time Employees Needed
11:00am-Noon	8
Noon-1:00pm	8
1:00pm-2:00pm	7
2:00pm-3:00pm	1
3:00pm-4:00pm	2
4:00pm-5:00pm	1
5:00pm-6:00pm	5
6:00pm-7:00pm	10
7:00pm-8:00pm	10
8:00pm-9:00pm	6
9:00pm-10:00pm	6

Let
x_1 = the number of part-time employees starting a four-hour shift at 11:00 am
x_2 = the number of part-time employees starting a four-hour shift at Noon
:
x_8 = the number of part-time employees starting a four-hour shift at 6:00pm

Part-time employees are paid an average of $7.60 per hour and each of them works a four hour shift; so the daily cost of part-time help is $4(7.60)($x_1 + x_2 + \ldots + x_8$)

Our Model:

Minimize $\sum_{i=1}^{8} 30.40x_i$

s.t. $x_1, x_2, \ldots, x_8 \geq 0$ and

$$
\begin{aligned}
x_1 &\geq 8\\
x_1 + x_2 &\geq 8\\
x_1 + x_2 + x_3 &\geq 7\\
x_1 + x_2 + x_3 + x_4 &\geq 1\\
x_2 + x_3 + x_4 + x_5 &\geq 2\\
x_3 + x_4 + x_5 + x_6 &\geq 1\\
x_4 + x_5 + x_6 + x_7 &\geq 5\\
x_5 + x_6 + x_7 + x_8 &\geq 10\\
x_6 + x_7 + x_8 &\geq 10\\
x_7 + x_8 &\geq 6\\
x_8 &\geq 6
\end{aligned}
$$

Employing the Management Scientist software we obtain an optimal solution of $x_1 = 8$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$, $x_5 = 2$, $x_6 = 0$, $x_7 = 4$, $x_8 = 6$ which optimizes the objective function at 608.

a) So a minimum-cost schedule for part-time employees is to have 8 such employees working from 11:00am-3:00pm, two such employees working from 3:00pm-7:00pm, four such employees working from 5:00pm-9:00pm, and six such employees working from 6:00pm-10:00pm.

b) Under the prescribed schedule the total daily payroll for part-time employees is $608 and four part-time shifts are needed. Since the prescribed schedule created a surplus of 16 employee hours each day, it looks like a desirable cost saving move would be to create some three-hour shifts.

Introducing the possibility of three-hour shifts leads to the creation of a revised model.

Let
x_9 = the number of part-time employees starting a three-hour shift at 11:00 am
x_{10} = the number of part-time employees starting a three-hour shift at Noon
:
x_{17} = the number of part-time employees starting a three-hour shift at 7:00pm

The daily cost of each four-hour employee is $30.40 and the daily cost of each three-hour employee is $22.80. So our revised model is as formulated below.

Minimize $\sum_{i=1}^{8} 30.40 x_i + \sum_{i=9}^{17} 22.80 x_i$

s.t. $x_1, x_2, \ldots, x_{17} \geq 0$ and

$$
\begin{aligned}
&x_1 &&+ x_9 &&\geq 8\\
&x_1 + x_2 &&+ x_9 + x_{10} &&\geq 8\\
&x_1 + x_2 + x_3 &&+ x_9 + x_{10} + x_{11} &&\geq 7\\
&x_1 + x_2 + x_3 + x_4 &&+ x_{10} + x_{11} + x_{12} &&\geq 1\\
&x_2 + x_3 + x_4 + x_5 &&+ x_{11} + x_{12} + x_{13} &&\geq 2\\
&x_3 + x_4 + x_5 + x_6 &&+ x_{12} + x_{13} + x_{14} &&\geq 1\\
&x_4 + x_5 + x_6 + x_7 &&+ x_{13} + x_{14} + x_{15} &&\geq 5\\
&x_5 + x_6 + x_7 + x_8 &&+ x_{14} + x_{15} + x_{16} &&\geq 10\\
&x_6 + x_7 + x_8 &&+ x_{15} + x_{16} + x_{17} &&\geq 10\\
&x_7 + x_8 &&+ x_{16} + x_{17} &&\geq 6\\
&x_8 &&+ x_{17} &&\geq 6
\end{aligned}
$$

Employing the Management Scientist software we obtain an optimal solution of
$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = 0$, $x_8 = 6$, $x_9 = 8$, $x_{10} = 0$, $x_{11} = 1$, $x_{12} = 0$, $x_{13} = 1$, $x_{14} = 0$, $x_{15} = 4$, $x_{16} = 0$, and $x_{17} = 0$ which optimizes the objective function at 501.60

c) In this case we need five part-time shifts and the associated daily cost of part-time workers is reduced to $501.60 – a daily savings of $106.40. The revised schedule is to have eight workers assigned to the three-hour 11:00am-2:00pm shift, one worker assigned to the three-hour 1:00pm-4:00pm shift, one worker assigned to the three-hour 3:00pm-6:00pm shift, four workers assigned to the three-hour 5:00pm-8:00pm shift, and six workers assigned to the four-hour 6:00pm-10:00pm shift.