

29. First we need to put the problem in standard form. We will introduce slack variables into constraints 1 and 2 and because constraint 3 is a strict equality we will introduce an artificial variable a_3 .

$$\begin{aligned} \max \quad & 120x_1 + 80x_2 + 14x_3 + 0s_1 + 0s_2 - Ma_3 \\ \text{s.t.} \quad & 4x_1 + 8x_2 + x_3 + 1s_1 + 0s_2 + 0a_3 = 200 \\ & 2x_2 + x_3 + 0s_1 + 1s_2 + 0a_3 = 300 \\ & 32x_1 + 4x_2 + 2x_3 + 0s_1 + 0s_2 + 1a_3 = 400 \\ & x_1, x_2, x_3, s_1, s_2, a_3 \geq 0 \end{aligned}$$

in Tableau form we get:

	x_1	x_2	x_3	s_1	s_2	a_3	
	120	80	14	0	0	-M	The initial basic solution is (0,0,0,200,300,400) which gives the of = -400M
s_1	4	8	1	1	0	0	200
s_2	0	2	1	0	1	0	300
a_3	-M	4	2	0	0	1	400
$C_j - Z_j$	120 - 4M	80 - 8M	14 - 2M	0	0	-M	-400M

After inserting the Tableau we see that we want to pivot in row 3 column 1. After pivoting we get the following Tableau (we bring x_1 into the basis and remove a_3 so we will eliminate the a_3 column from the Tableau).

	x_1	x_2	x_3	s_1	s_2	
	120	80	14	0	0	This solution is (12.5, 0, 0, 150, 300) and gives us 1500
s_1	0	15	1/4	1	0	250
s_2	0	2	1	0	1	300
x_1	120	1	1/8	0	0	25
$C_j - Z_j$	0	65	6.5	0	0	1500

cont. Inspecting the Tableau we see we want pivot in row 1 column 2, bringing x_2 into the basis and removing s_1 .

	x_1	x_2	x_3	s_1	s_2	
	120	80	14	0	0	This solution is (10, 20, 0, 0, 260)
x_2	80	0	1/10	1/15	0	20
s_2	0	0	4/5	1/15	1	260
x_1	120	1	1/20	1/10	0	10
	120	80	14	8.67	0	2800
$C_j - Z_j$	0	0	0	-8.67	0	

This is an optimal solution which gives us $120(10) + 20(80) = 2800$

When we look at the $C_j - Z_j$ row we see there is a zero in the x_3 column, which is a non-basic variable. This means we will have alternate optimal solutions. Pivoting in row 1 column 3 (bringing x_3 into the basis, removing x_2 we get the alternate optimal solution:

	x_1	x_2	x_3	s_1	s_2	
	120	80	14	0	0	This gives a solution (0, 0, 200, 0, 100)
x_3	14	0	10	1/5	0	200
s_2	0	0	-8	-2/5	1	100
x_1	120	1	1/2	1/10	0	0
	120	80	14	8.67	0	2800
$C_j - Z_j$	0	0	0	-8.67	0	

This gives the objective function the value: $(0)(120) + (0)(80) + 200(14) = 2800$ so we see it is an alternate optimal solution.