We will consider the ten sections as a single unit. We can do this because we will assume the conditions are the same for each section and the relevant processes are linear satisfying the assumptions of L.P. models.

Let xc = the number of cattle raised
xg = the number of goats raised
xs = the number of sheep raised
xd = the number of deer raised
xt = the number of turkeys raised
xf = the number of fish raised

In our final solution we can think of 1/10 of each xi (i = c, g, s, d. t, f) assigned to each section.

The respective relative monetary income values per animal, animal unit equivalences (measures environmental impact) per animal, and percent of each species that can be harvested each year are displayed in the table below.

Animal Species	Relative Income Value	Animal Unit Equivalent	Harvest % to Maintain Population
Cattle	10	1	25
Goats	1	0.25	NA
Sheep	1	0.2	35
Deer	0.5	0.3	15
Turkey	0.05	0	20
Fish	0.001	0	25

Our constraints are as follows.

To organize his operation for livestock production the farmer must have at least 20 cattle and at least 20 goats on his ranch. This condition leads to the constraints

(1) $xc \geq 20$ and (2) $xg \geq 20$.

The farmer says he wants some sheep and deer. If he chooses to pursue that desire then we would seem to have the constraints

(3) $xs \geq 1$ and (4) $xd \geq 1$.

However, if he wishes to harvest sheep and deer and maintain both populations, then we would have the constraints

(3’) $xs \geq 3$ and (4’) $xd \geq 7$.

If the farmer wants to raise turkeys and fish, then he must raise (a) cattle, sheep, goats, and deer, (b) cattle, sheep, and goats, or (c) cattle goats, and deer, but no more than 75% of the grazing load may be due to just cattle and goats. Each section has a maximum carrying capacity of 10 animal units per year; so the ranch's maximum carrying capacity is 100 animal units. So, the 75% load due to cattle and goats constraint can be expressed as

(5) $1xc + 0.25xg \leq 75$.

If the farmer elects to raise turkeys and fish on each section, then he would need to satisfy condition (a), (b), or (c) above on each section. Constraints (1) and (2) guarantee that he will have enough cattle and goats. In addition, at least one of the following conditions would need to be satisfied

(3'') $xs \geq 10$ or (4'') $dx \geq 10$.

Also, we have the conditions restricting the numbers of fish and turkeys that can be rasied on the farm.

(6) $xt \leq 200$ and (7) $xf \leq 5000$

In any case, turkeys and fish or not, we have the constraint that the maximum carrying capacity of the ranch is 100 animal units.

(7) $1xc + 0.25xg + 0.2xs + 0.3xd \leq 100$.

We will consider two cases determined by whether or not the farmer elects to raise turkeys and fish.

In both cases our revenue function is given by

$$R = 2.5xc + 1.0xg + 0.35xs + 0.075xd + 0.01xt + 0.00025xf$$

Case I. Farmer Raises Turkey and Fish

In this case we will assume that "some" sheep and deer means at least ten of each.

Here is the L.P. problem for this case.

Maximize $R = 2.5xc + 1.0xg + 0.35xs + 0.075xd + 0.01xt + 0.00025xf$

subject to

$xc \geq 20$, $xg \geq 20$, $xs \geq 10$, $xd \geq 10$, $xt \leq 200$, $xf \leq 5000$, $1xc + 0.25xg \leq 75$,
$1xc + 0.25xg + 0.2xs + 0.3xd \leq 100$, $xt \geq 0$, and $xf \geq 0$.

Case II. Farmer Raises Neither Turkeys Nor Fish

We could subdivide this case into two subcases. Either farmer elects to raise sheep and dear or not. In both subcases we may eliminate constraint (5). If the farmer elects to raise sheep and deer we will interpret "some" via constraints (3') and (4').

Subcase A. Farmer Raises Sheep and Deer

In this case our L.P. problem is the following.

Maximize $R = 2.5xc + 1.0xg + 0.35xs + 0.075xd + 0.01xt + 0.00025xf$

subject to

$xc \geq 20$, $xg \geq 20$, $xs \geq 3$, $xd \geq 7$, $xt = 0$, $xf = 0$, and
$1xc + 0.25xg + 0.2xs + 0.3xd \leq 100$.

Subcase B. Farmer Raises Neither Sheep Nor Deer

In this case our L.P. problem is the following.

Maximize $R = 2.5xc + 1.0xg + 0.35xs + 0.075xd + 0.01xt + 0.00025xf$

subject to

$xc \geq 20$, $xg \geq 20$, $xs = 0$, $xd = 0$, $xt = 0$, $xf = 0$, and
$1xc + 0.25xg \leq 100$.

Solution for Case I. (Using Management Science Software)

$xc = 20$, $xg = 220$, $xs = 110$, $xd = 10$, $xt = 200$, $xf = 5000$ and $R = 312.50$

Solution for Case IIA. (Using Management Science Software)

$xc = 20$, $xg = 309$, $xs = 3$, $xd = 7$, $xt = 0$, $xf = 0$ and $R = 360.78$

Solution for Case IIB. (Using Management Science Software)

$xc = 20$, $xg = 320$, $xs = 0$, $xd = 0$, $xt = 0$, $xf = 0$ and $R = 370.00$

Discussion, Conclusions, and Recommendations

The environmentally friendly approach would be to act under the model of Case I. That is, raise 20 cattle/2 per section, 220 goats/22 per section, 110 sheep/11 per section, 10 deer/1 per section, 200 turkeys/20 per section, and 5000 fish/500 per section. That allocation yields 312.50 revenue units per year. This is the recommended approach because the allocation can be maintained in the long run with minimal environmental degradation.

Of course, in Case IIB revenue can be maximized at 370 revenue units if the farmer elects to raise 20 cattle/2 per section, 320 goats/32 per section and neither sheep, deer, turkeys, nor fish. This allocation will probably not maintain the desired vegetation cover in the long run.

If the farmer agrees to raise neither turkeys nor fish but still wishes to raise at least a token number of sheep and deer (sufficient to harvest one each per year), we have Case IIA. In this case 20 cattle/2 per section, 309 goats/about 31 per section, 3 sheep and 7 deer will yield 360.78 revenue units per year. It is unlikely that this allocation will maintain the ecological balance we seek in the long run.

Sensitivity Analysis of Recommended Case I

Revenue can be increased if the right hand sides of constraints (5) and (7), [Constraints 1 and 2 in the attached output] could be increased. That does not appear to be an option since those numbers are not subject to revision. Small percentage changes in the objective function coefficients will not affect the solution. However, the dual prices for constraints (1) and (4) [3 and 6 in the attached output] suggest that allowing fewer cattle and deer will lead to improved revenue. That later option should be discussed with the farmer.

If one assumes that revenue is gained from "only" one-half the goat population, we would have different optimal solutions under the cases we considered.

```
MAX 2.5xc+1xg+0.35xs+0.075xd+0.01xt+0.00025xf
     S.T.

        1)  1xc+0.25xg<75
        2)  1xc+0.25xg+0.2xs+0.3xd<100
        3)  1xc>20
        4)  1xg>20
        5)  1xs>10
        6)  1xd>10
        7)  1xt<200
        8)  1xf<5000
```

OPTIMAL SOLUTION
Objective Function Value = 312.50000

Variable	Value	Reduced Costs
xc	20.00000	0.00000
xg	220.00000	0.00000
xs	110.00000	0.00000
xd	10.00000	0.00000
xt	200.00000	0.00000
xf	5000.00000	0.00000

Constraint	Slack/Surplus	Dual Prices
1	0.00000	2.25000
2	0.00000	1.75000
3	0.00000	-1.50000
4	200.00000	0.00000
5	100.00000	0.00000
6	0.00000	-0.45000
7	0.00000	0.01000
8	0.00000	0.00025

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
xc	No Lower Limit	2.50000	4.00000
xg	0.62500	1.00000	No Upper Limit
xs	0.05000	0.35000	0.80000
xd	No Lower Limit	0.07500	0.52500
xt	0.00000	0.01000	No Upper Limit
xf	0.00000	0.00025	No Upper Limit

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
1	25.00000	75.00000	95.00000
2	80.00000	100.00000	No Upper Limit
3	0.00000	20.00000	70.00000
4	No Lower Limit	20.00000	220.00000
5	No Lower Limit	10.00000	110.00000
6	0.00000	10.00000	76.66667

7	0.00000	200.00000	No Upper Limit
8	0.00000	5000.00000	No Upper Limit