1. Job Assignment Problem (20 points)

In this case we have a set of applicants $A = \{A_1, A_2, A_3, A_4\}$ and a set of jobs $J = \{J_1, J_2, J_3, J_4\}$. A_i's rating (qualifications) for J_k is given by r_{ik} where r_{ik} is the (i,k)-enrty of the matrix R.

$$R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 3 & 3 & 5 & 0 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & 3 & 2 \end{bmatrix}$$

We assume that more qualified applicants are given higher ratings. Minimally qualified applicants are given a rating of 1; unqualified applicants are given a rating of zero.

We introduce the variables $x_{ik} = \{1 \text{ if } A_i \text{ is assigned to } J_k \text{ and } 0 \text{ otherwise}\}$ where i = 1, 2, 3, 4; j = 1, 2, 3, 4.

We assume that each applicant is assigned to at most one job and each job is filled by exactly one qualified applicant.

Our L.P. problem can be formulated as follows.

$$\text{Max } p = 1x_{12}+ x_{13}+1x_{14}+3x_{21}+3x_{22}+5x_{23}+2x_{31}+1x_{32}+1x_{33}+1x_{34}+3x_{41}+2x_{42}+3x_{43}+2x_{44}$$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} \leq 1$ (A_1 is assignment to at most one job.)
$x_{21} + x_{22} + x_{23} + x_{24} \leq 1$ (A_2 is assignment to at most one job)
$x_{31} + x_{32} + x_{33} + x_{34} \leq 1$ (A_3 is assignment to at most one job)
$x_{41} + x_{42} + x_{43} + x_{44} \leq 1$ (A_4 is assignment to at most one job)
$x_{11} + x_{21} + x_{31} + x_{41} = 1$ (J_1 is awarded to exactly one applicant)
$x_{12} + x_{22} + x_{32} + x_{42} = 1$ (J_2 is awarded to exactly one applicant)
$x_{13} + x_{23} + x_{33} + x_{43} = 1$ (J_3 is awarded to exactly one applicant)
$x_{14} + x_{24} + x_{34} + x_{44} = 1$ (J_4 is awarded to exactly one applicant)
$x_{ij} \geq 0$ for all $1 \leq i, j \leq 1$

We may have to include the additional constraints $x_{11} = 0$, $x_{13} = 0$, and $x_{24} = 0$ to insure that unqualified applicants are not inappropriately assigned to jobs for which they are not qualified. If such infeasible assignments are made by our algorithm, we will add those three constraints and re-run the algorithm.

Management Scientist Solution

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OPTIMAL SOLUTION:  Objective Function Value = 10.000
      Variable             Value
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        x11                  0.000
        x12                  1.000
        x13                  0.000
        x14                  0.000
        x21                  0.000
        x22                  0.000
        x23                  1.000
        x24                  0.000
        x31                  1.000
        x32                  0.000
        x33                  0.000
        x34                  0.000
        x41                  0.000
        x42                  0.000
        x43                  0.000
        x44                  1.000
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Assign A_1 to J_2; Assign A_2 to J_3; Assign A_3 to J_1; Assign A_4 to J_4. The value of assignments is 10. That assignment is not unique in producing the optimal value of 10. An alternative would be to assign A_1 to J_4; Assign A_2 to J_3; Assign A_3 to J_2; Assign A_4 to J_1. (Neither assignment results in an applicant being assigned to a job for which he/she is not qualified.)

Although it is likely that our assumptions that (a) a rating of zero indicates an unqualified applicant, (b) no applicant can be assigned no more than one job, and (c) each job is assigned to exactly one applicant are valid, it is possible that one or more of those assumptions will not be true in a particular situation requiring an alternative model.