

## Pollution on the Peacock – A Concise Statement of Relevant Facts

PPP, Inc. produces two types of paper – Type A yields a profit of \$10/ton  
 Type B yields a profit of \$7/ton

Facilities limit production to 200,000 tons per year of which at most 100,000 tons can be of type A.

Operating at an optimal rate, PPP, Inc. can have a net profit of \$1,700,000/year.

2000 gallons of water is used in the production of each ton of type A paper  
 1000 gallons of water is used in the production of each ton of type B paper

Water used in the production of type A paper is returned to the river with a residue of 20 pounds of noxious chemicals per 1000 gallons of water. Water used in the production of type B paper is returned to the river with a residue of 25 pounds of noxious chemicals per 1000 gallons of water.

New environmental regulations are being imposed. If “x” denotes the number of pounds of noxious chemicals being returned to the river per 1000 gallons of water, for  $x > 3$ , then a fine of  $\$(x - 3)/10$  per 1000 gallons used is assessed. Also, no more than 250,000,000 gallons of water may be used per year.

PPP, Inc. can reduce the chemical content of the water being returned to the river by several types of treatment. Water can be treated after use by installing mechanical filters or chemical precipitators on the outflow lines from the production facilities of each type of paper independently. Alternatively, PPP, Inc. could build a sedimentation lagoon which would treat the outflow from both production facilities simultaneously. Data regarding these options are given below. The “Content” column gives the chemical content of the treated outflow water from each production unit. The “Cost” column gives the cost of installing and operating the various treatment methods for production of each type of paper.

Type of Treatment	Content (lb) (per 1000 gal)		Cost (\$) (per 1000 gal)	
	Type A	Type B	Type A	Type B
Mech filter	11	12	0.60	0.50
Chemical precipitator	5	8	1.00	0.60
High-efficiency chemical precipitator	4	7	1.10	0.90
Sedimentation lagoon	3	3	*	*

The cost of building a sedimentation lagoon is \$1.5 million and will be adequate for treating all the water used for 5 years after which time it will be filled. So, if a lagoon is built it will be amortized at a rate of \$300,000 per year.

*Problem 1.* Which treatment devices (if any) should be installed and how should production be allocated between paper of types A and B to maximize profit?

*Problem 2.* Formulate and interpret the dual of the maximization of the problem posed in Problem 1.

*Problem 3.* How could we make our model more realistic?