Pollution on the Peacock Case Study (Continued)

*Understanding and Defining the Problem*

As a first step in coming to grips with this problem we will see if we can determine how PPP, Inc. can earn an optional profit of $1,700,000 per year in the absence of environmental controls and sanctions. If no water treatment methods are applied and no fines or restrictions on water use are imposed then we can use the following model.

Let XA = the number of tons of type A paper produced in a year,
XB = the number of tons of type B paper produced in a year, and
P = the annual profit, in dollars, from the production of types A and B paper.

In this case our problem is to

Maximize $P = 10XA + 7XB$ subject to $XA, XB \geq 0$ and

$$XA + XB \leq 200{,}000$$
$$XA \leq 100{,}000$$

In this case our optimal solution is XA = ________, XB = ________, and P = _________.

Of course, the problem of interest will have a third structural constraint due to the restriction on the total number of gallons of water that can be used by PPP, Inc. in one year. That constraint can be written as

$$2000XA + 1000XB \leq 250{,}000{,}000 \text{ or as } 2XA + 1XB \leq 250{,}000$$

The objective function must also be modified to account for the reduction in profit due to the cost of the treatment methods and the fines assessed. Of course, the revised coefficients for XA and XB in the objective function will depend on which treatment method we apply in the production of each type of paper. Mechanical and chemical precipitators can be applied independently to the outflow pipes for the production facilities for type A and type B paper, but under the sedimentation lagoon approach, all water used flows into the lagoon. Consequently, *we will start by considering the use of treatment methods without using a lagoon.* Subsequently we will consider the use of a lagoon.

In the absence of a lagoon, suppose $C_1$ is the profit per ton for type A paper produced under a particular type of treatment and $C_2$ is the profit per ton for type B paper produced under a particular treatment method. *Assuming at most one treatment method is applied to each production process*, our problem will be to

Maximize $P = C_1XA + C_2XB$ subject to $XA, XB \geq 0$ and

$$XA + XB \leq 200{,}000$$
$$XA \leq 100{,}000$$
$$2XA + 1XB \leq 250{,}000$$

In what follows we will determine the values of $C_1$ and $C_2$ for the various alternative treatment methods.

*Identify the Alternatives*

If we were to relax our assumption that at most one treatment method is applied to each production process, we could subject some of the production of each type of paper to each of the treatment alternatives. In that case we would have eight decision variables rather than just two.

Let XA0 = the number of tons of type A paper produced using no treatment method,
XB0 = the number of tons of type B paper produced using no treatment method,
XA1 = the number of tons of type A paper produced using a mechanical filter,
XB1 = the number of tons of type B paper produced using a mechanical filter,
XA2 = the number of tons of type A paper produced using a chemical filter,
XB2 = the number of tons of type B paper produced using a chemical filter,
XA3 = the number of tons of type A paper produced using a high efficiency filter, and
XB3 = the number of tons of type B paper produced using a high efficiency filter.

The table below gives the implementation costs per ton of paper produced and the associated fines per ton of paper produced for each type of treatment and each type of paper. We will also include the option of no treatment control.

| Type of Treatment | Implementation Costs($)/ton Produced | | Fine($)/ton Produced | |
|---|---|---|---|---|
| | Type A | Type B | Type A | Type B |
| Mechanical Filter | | | | |
| Chemical precipitator | | | | |
| High-efficiency chemical precipitator | | | | |
| No control | | | | |

The profit per ton for each type of paper produced under the three types of treatment above is given in the next table. With this information we can determine the coefficients of our decision variables in the objective function. We can then form a maximization LP problem.

| Type of Treatment | Profit/ton of Type A | Profit/ton of Type B |
|---|---|---|
| Mechanical Filter | | |
| Chemical precipitator | | |
| High-efficiency chemical precipitator | | |
| No control | | |

*Determine the Criteria*

We have two possible LP models without considering a lagoon and also the case of a lagoon.

*Choose an Alternative*