Math 460 Session #17

A Standard Form of the LP Model

Choose $x_1, x_2, \ldots, x_n$ so as to

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to

$$a_{11} + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21} + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1} + a_{m2} + \ldots + a_{mn}x_n = b_m,$$

and $x_1, x_2, \ldots, x_n \geq 0.$

Other Forms

Minimizing rather than maximizing the objective function
Some constraints with greater-than-or-equal-to inequality
Some constraints in less-than-or-equal-to form
Deleting nonnegativity constraints for some decision variables

Some Terminology

A **feasible solution** is a solution for which *all* the constraints are satisfied.
An **optimal solution** is a feasible solution that has the *most favorable associated value* of the objective function.
A **basic solution** is a solution with no more than m of the variables assigned nonzero values. (Those variables are basic variables.)
A **basic feasible solution** is a feasible solution with no more than m of the variables assigned positive values.
A **degenerate basic feasible** solution has fewer than m of the variables assigned positive values. **A nondegenerate basic feasible** solution is a basic feasible solution with exactly m variables assigned positive values.

Assumptions of LP

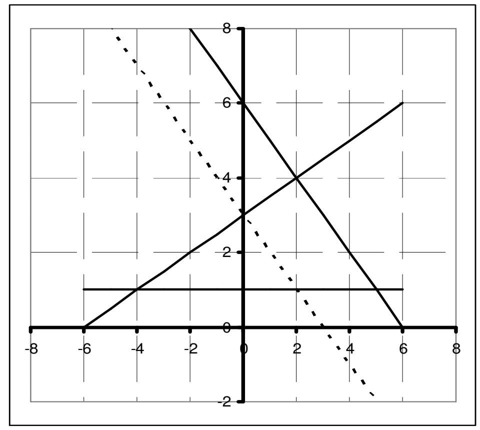
Proportionality – The contribution to the objective function and the amount of resources used in each constraint are proportional to the value of each decision variable.
Additivity – The value of the objective function and the total resources used can be found by summing the objective function contribution and the resources used for all decision variables.
Divisibility – The decision variables are continuous.
Certainty – The parameters of the model (the a_{ij}, b_i, and c_j) are known constants.

The Assumptions in Perspective

Mathematical models are *idealized representations* of real problems (situations).
In many real applications *almost none* of the assumptions will hold completely.

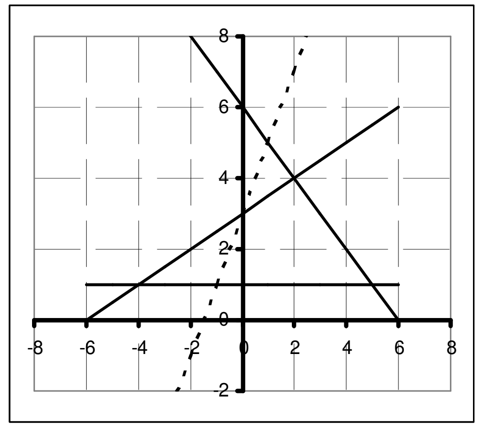
Special Cases

1.
Maximize $Z = 2x_1 + 2x_2$
st
$$1x_1 + 1x_2 \le 6$$
$$-1x_1 + 2x_2 \le 6$$
$$1x_2 \ge 1,$$
and $x_1, x_2 \ge 0$.



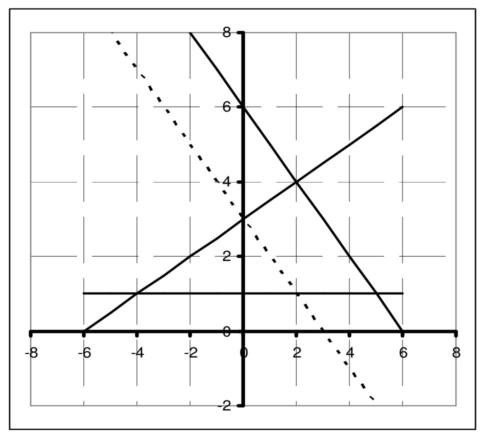
2.
Maximize $Z = -2x_1 + 1x_2$

st
$$1x_1 + 1x_2 \le 6$$
$$-1x_1 + 2x_2 \le 6$$
$$1x_2 \ge 1,$$
and $x_1, x_2 \ge 0$.



3.
Maximize $Z = 2x_1 + 2x_2$
st

$$1x_1 + 1x_2 \le 6$$
$$-1x_1 + 2x_2 \ge 6$$
$$1x_2 \le 1,$$
and $x_1, x_2 \ge 0$.



4.
Maximize $Z = 2x_1 + 2x_2$
st

$$1x_1 + 1x_2 \ge 6$$
$$-1x_1 + 2x_2 \ge 6$$
$$1x_2 \ge 1,$$
and $x_1, x_2 \ge 0$.