

**Session 19 – Example of Simplex Based Sensitivity Analysis**

**Maximize  $P = 2x_1 + 3x_2$**

**s.t.  $x_1, x_2 \geq 0$  and**

**$1x_1 + 2x_2 \leq 6$**

**$2x_1 + 1x_2 \leq 8$**

**Initial Simplex Tableau:**

<i>Basis</i>						<b>RHS</b>
		$x_1$	$x_2$	$s_1$	$s_2$	
	$c_B$	<b>2</b>	<b>3</b>	<b>0</b>	<b>0</b>	
$s_1$	<b>0</b>	<b>1</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>6</b>
$s_2$	<b>0</b>	<b>2</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>8</b>
$z_j$		<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
$c_j - z_j$		<b>2</b>	<b>3</b>	<b>0</b>	<b>0</b>	

**The Final Tableau:**

<i>Basis</i>						<b>RHS</b>
		$x_1$	$x_2$	$s_1$	$s_2$	
	$c_B$	<b>2</b>	<b>3</b>	<b>0</b>	<b>0</b>	
$x_2$	<b>3</b>	<b>0</b>	<b>1</b>	<b>2/3</b>	<b>-1/3</b>	<b>4/3</b>
$x_1$	<b>2</b>	<b>1</b>	<b>0</b>	<b>-1/3</b>	<b>2/3</b>	<b>10/3</b>
$z_j$		<b>2</b>	<b>3</b>	<b>4/3</b>	<b>1/3</b>	<b>32/3</b>
$c_j - z_j$		<b>0</b>	<b>0</b>	<b>-4/3</b>	<b>-1/3</b>	

**Optimal Solution:  $x_1 = 10/3, x_2 = 4/3, P = 32/3$**

We calculate the range of optimality for the objective function coefficient of  $x_1$  by replacing the numerical value of the coefficient of  $x_1$  with  $c_1$  everywhere it occurs in the final simplex tableau.

<i>Basis</i>						<b>RHS</b>
		$x_1$	$x_2$	$s_1$	$s_2$	
	$c_B$	$c_1$	<b>3</b>	<b>0</b>	<b>0</b>	
$x_2$	<b>3</b>	<b>0</b>	<b>1</b>	<b>2/3</b>	<b>-1/3</b>	<b>4/3</b>
$x_1$	$c_1$	<b>1</b>	<b>0</b>	<b>-1/3</b>	<b>2/3</b>	<b>10/3</b>
$z_j$		$c_1$	<b>3</b>	$2 - c_1/3$	$-1 + 2c_1/3$	$4 + 10c_1/3$
$c_j - z_j$		<b>0</b>	<b>0</b>	$c_1/3 - 2$	$-2c_1/3 + 1$	

$x_1 = 10/3, x_2 = 4/3$  will remain the optimal solution provided

$c_1/3 - 2 \leq 0$  and  $-2c_1/3 + 1 \leq 0$ .

Finish the computation of the range of optimality for the objective function coefficient of  $x_1$  in the space below.